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FAKULTÄT FÜR MATHEMATIK
Dekan Univ.-Prof. Dr. Radu Ioan Boţ

Einladung zur öffentlichen Defensio

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Thema der Dissertation

Topics in the Theory of Localized Frames

Abstract:

This doctoral thesis consists of three papers in frame theory. Frames are countable families of vectors in a separable Hilbert space, which enable a bounded, linear, and stable reconstruction of any vector in that space from its frame coefficients. A natural generalization of frames are operator-valued frames, which analogously provide perfect reconstruction from linear higher-rank measurements. In this thesis, we study (operator-valued) frames whose elements satisfy an additional incoherence property called localization. The quality of localization is measured by the o"-diagonal decay of the (operator-valued) Gram matrix of the underlying (operator-valued) frame. More precisely, such a frame is called localized, if its associated Gram matrix belongs to some suitable inverse-closed matrix algebra.

The first paper is devoted to the study of algebras of operator-valued matrices, which are inverse-closed in the Banach algebra of bounded operators acting on the Bochner space of square-summable Hilbert space-valued sequences. In particular, we study operator-valued versions of weighted Schur-type algebras, the Jaffard algebra and generalizations of it, the Baskakov-Gohberg-Sjöstrand algebra, and their anisotropic versions, and show that each of these matrix algebras is inverse-closed and, in fact, a symmetric Banach algebra.

With the matrix algebras from the first paper in mind, we introduce an abstract localization concept for operator-valued frames in the second paper. We prove that intrinsic localization of an operator-valued frame is preserved by its canonical dual. We show that for any localized operator-valued frame, there is a whole family of associated (quasi-) Banach spaces attached to it. We prove that the series associated with perfect reconstruction of the given operator-valued frame converges not only in the underlying Hilbert space but also in each of these spaces. Finally, we apply our theory to irregular Gabor g-frames.

In the third paper, we prove the equivalence of the frame property of a given suitably localized sequence in a Hilbert space and nine other conditions that do not involve an

inequality. This result is applied in the context of shift-invariant spaces, where we obtain new conditions for stable sets of (irregular) sampling.

Prüfungssenat

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