universität wien

FAKULTÄT FÜR MATHEMATIK
Dekan Univ.Prof. Dr. Radu Ioan Bot

Einladung zur öffentlichen Defensio

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Thema der Dissertation

Definable Witnesses

Abstract:
The topic of this doctoral work is at the intersection of descriptive set theory, set theory of the reals, and forcing. We study methods of obtaining models of ZFC plus the negation of the continuum hypothesis in which there exist various combinatorially significant subsets of the real line that are definable by a projective formula of provably minimal complexity, i.e. is optimal. We present a general framework for obtaining the existence of coanalytic combinatorial sets of reals given the existence of a Σ^1_2 such witness. This is done by collecting a series of these reduction theorems appearing sporadically throughout the literature, extracting from their proofs an overarching pattern, and presenting them in a uniform fashion. These theorems improve previous constructions which required the assumption $V = L$. The general strategy is applied to provide a new reduction theorem for the case of Hausdorff gaps. We then study forcing notions relevant to the theory of cardinal characteristics; first we show a proper forcing given by Shelah in 1984 and its countable support iterations preserve tight mad families, providing an alternative proof of the consistency of $\mathfrak{b} = \mathfrak{a} < \mathfrak{s}$. This is applied to show that $\mathfrak{a} < \mathfrak{s}$ is consistent with a Δ^1_3 wellordering of the reals and a coanalytic tight mad family witnessing $\mathfrak{a} = \aleph_1$, responding to questions of Fischer and Friedman. We extend the theory of projective witnesses for values of cardinal characteristics by considering the spectrum of $\mathfrak{a}$, the set of possible cardinalities of a mad family. By showing a forcing notion of Friedman and Zdomskyy also preserves tight mad families in a strong sense, we obtain the consistency of $\mathfrak{a} = \aleph_1 < \mathfrak{c} = \aleph_2$, and for every κ in the spectrum of $\mathfrak{a}$ there exists a

projective tight mad family of size κ with definition of optimal complexity. In the end we show the compatibility of our results and construct a model witnessing all of the conclusions simultaneously.

Prüfungssenat

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Zeit und Ort

Freitag, 17. Oktober 2025, 16:00 Uhr

Online:

https://univienna.zoom.us/j/62138828442?pwd=2Rygzfl1JQawEJNcc3SojOmwCEbc5A.1

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