



Einladung zur öffentlichen Defensio

Stefan Christian SCHROTT

Thema der Dissertation

Stochastic Mass Transport: From Discrete to Continuous Time

Abstract:

This thesis is concerned with probabilistic variants of the transport problem, in particular weak optimal transport and adapted optimal transport.

Weak optimal transport was introduced by Gozlan, Roberto, Samson and Tetali as a nonlinear relaxation of classical optimal transport. On the one hand, this framework of weak optimal transport still retains many characteristics of usual optimal transport, allowing for a compelling theory. On the other hand, this type of relaxation is suitable to cover a number of problems that lie outside the scope of the classical theory such as entropic optimal transport, martingale optimal transport and transport problems with barycentric costs.

In [12] we establish a general duality theorem for weak optimal transport together with dual attainment as well as complementary slackness conditions which characterize primal and dual optimizers, i.e. we establish a fundamental theorem (in the sense of [4, Theorem 1.13]).

As applications we provide concise derivations of the Brenier–Strassen theorem, the convex Kantorovich–Rubinstein formula and the structure theorem of entropic optimal transport. We also extend Strassen’s theorem in the direction of Gangbo–McCann’s transport problem for convex costs. Moreover, we determine the optimizers for a new family of transport problems which contains the Brenier–Strassen, the martingale Benamou–Brenier and the entropic martingale transport problem as extreme cases.

The second main topic of the thesis is adapted optimal transport and adapted weak topologies. Researchers from different areas have independently defined extensions of the usual weak convergence of laws of stochastic processes with the goal of adequately accounting for the flow of information. Natural approaches are convergence of the Aldous–Knight prediction process, Hellwig’s information topology, convergence in adapted distribution in the sense of Hoover–Keisler, and the weak topology induced by optimal stopping problems.

A particular contribution of this thesis is that on continuous processes with natural filtrations there exists a canonical adapted weak topology which can be defined by all of these approaches; moreover, the adapted weak topology is metrized by a suitable adapted Wasserstein distance $\mathcal{AW}$.

While the set of processes with natural filtration is not complete, we establish that its completion consists precisely of the space $\mathcal{FP}$ of stochastic processes with general filtrations. We also show that $(\mathcal{FP}, \mathcal{AW})$ exhibits several desirable properties. Specifically, it is Polish, martingales form a closed subset, and optimal stopping problems are continuous with respect to $\mathcal{AW}$.

Furthermore, we establish various results which concern adapted transport in discrete time and the connection between the discrete time and continuous time case. In particular, we show that adapted Monge couplings are dense in adapted couplings, thereby establishing that adapted Kantorovich problem is the relaxation of the adapted Monge problem. Moreover, we establish a Skorokhod representation result for the adapted Wasserstein distance: A sequence of laws converge in the adapted Wasserstein distance if and only if there exists a filtered probability space and processes on it that have the respective adapted distributions and converge pointwise. Finally, we connect the discrete and continuous time frameworks by strengthening results such as Donsker's theorem from weak convergence to adapted weak convergence.

Prüfungssenat

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Zeit und Ort

Dienstag, 30. September 2025, 14:00 Uhr

Online:

<https://univiennea.zoom.us/j/63409418072?pwd=OSaDeT3oKernMcNC7koIT7yHfX02FG.1>

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