universität wien

FAKULTÄT FÜR MATHEMATIK
Dekan Univ.Prof. Dr. Radu Ioan Bot

Einladung zur öffentlichen Defensio

Moritz DOBER

Thema der Dissertation

Phase transitions in planar statistical mechanics: Ashkin–Teller and related models

Abstract:

This thesis is devoted to the study of phase transitions and the associated behaviour in the Ashkin–Teller (AT) model on the square lattice $\mathbb{Z}^2$ and its related models, including the six- and eight-vertex models, the Potts model and FK percolation. The AT model was introduced in 1943 as a generalisation of the Ising model and may be represented by a pair of Ising spin configurations with coupling constants J, J' for each and U for their pointwise product, the latter describing the interaction within the pair. The thesis can be divided into three parts.

The first part is a study of the isotropic ($J = J'$) ferromagnetic AT model on the square lattice $\mathbb{Z}^2$. We confirm the presence of a *single phase transition* when $J \geq U > 0$ and *two distinct* ones when $U > J > 0$. In the uniqueness case, we identify the transition at the self-dual curve. In the non-uniqueness case, we show that the two distinct transition curves are dual to each other. A significant part is the detailed study of the *self-dual* model via its relations to the six-vertex model and self-dual FK percolation. The main result in this regard is *exponential decay* of correlations of the single spins and *uniform positivity* of correlations of the product, each in finite volume and under the respective least favourable boundary conditions, when $U > J$.

In the second part, we investigate the AT model in the presence of negative coupling constants on the hypercubic lattices $\mathbb{Z}^d$ in any dimension $d \geq 2$. Observing that it suffices to study the case $J, J' \geq 0 > U$, we confirm both *ferromagnetic and antiferromagnetic*

behaviour in this regime. We first establish the existence of a partial antiferromagnetic phase in the isotropic model in a perturbative sub-regime. In $d = 2$, we show that the associated staggered six-vertex *height function* is *localised,* although it simultaneously exhibits antiferromagnetic behaviour. We then demonstrate ferromagnetic behaviour by partitioning another sub-regime into a collection of smooth curves along which the model undergoes a subcritically *sharp order-disorder phase transition,* circumventing the difficulty of the lack of general monotonicity properties in the parameters.

The third part builds on the first one and deals with the associated self-dual Potts model with $q > 4$ states on the square lattice $\mathbb{Z}^2$. Under *order-disorder Dobrushin* boundary conditions on a square box of size n, we verify that the *interface* between the ordered and disordered phases is thin, has *fluctuations* of order $\sqrt{n}$ and converges when rescaled to a *Brownian bridge*. This is achieved by a coupling with a graphical representation of the AT model, which allows to relate the interface in the Potts model to a subcritical cluster conditioned to be long, and then developing the celebrated Ornstein–Zernike theory to deduce convergence of the latter. We also show the analogous statements for self-dual FK percolation with $q > 4$. In a subsequent work, which is not part of the thesis, we further build on this project and establish the so-called *wetting phenomenon* in the Potts model. We present the relevant coupling of the Potts model under order-order Dobrushin boundary conditions and a graphical representation of the AT model, conditioned to admit a pair of long clusters, which turn out to be repulsive to each other.

Prüfungssenat

Univ.-Prof. Mag. Dr. Andreas Cap
(Vorsitz, Universität Wien)

Assoz.-Prof. Alexander Glazman, PhD
(Universität Innsbruck)

Prof. Dr. Yvan Velenik
(Université de Genève)

Prof. Dr. Gábor Pete
(Budapest University of Technology and Economics)

Zeit und Ort

Donnerstag, 5. Juni 2025, 14:00 Uhr

Seminarraum 1, Erdgeschoß, Oskar-Morgenstern-Platz 1