Badly approximable vectors and best approximation

Nikolay Moshchevitin

One of the definitions of a badly approximable number α is related to its continued fraction expansion

$$\alpha = \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cdots}}}, \quad a_j \in \mathbb{Z}_+.$$

Real irrational α is badly approximable if its partial quotients in continued fraction expansion are bounded, that is $\sup_\nu a_\nu < \infty$. Define q_ν to be the denominator of ν-th convergent p_ν/q_ν to α and $\xi_\nu = |q_\nu \alpha - p_\nu|$. From the well known formula $q_\nu = a_\nu q_{\nu-1} + q_{\nu-2}$ we immediately deduce the equivalence

$$\alpha \text{ is badly approximable} \iff \sup_\nu \frac{q_{\nu+1}}{q_\nu} < \infty \iff \inf_\nu \frac{\xi_{\nu+1}}{\xi_\nu} > 0.$$

In our lecture we will discuss how these equivalences look like in the multidimensional settings related to simultaneous approximations and, more generally, systems of linear forms. Our talk is based on recent results of the speaker obtained jointly with R. Akhunzhanov, A. Marnat and J. Schleischitz as well as two recent paper by W.M. Schmidt and L. Summerer.