

## Seminar talk

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University of Vienna, Faculty of Mathematics, Oskar-Morgenstern-Platz 1  
Room BZ09 (9th floor, Besprechungszimmer)

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## Lucas congruences and congruence schemes

It is a well-known and beautiful classical result of Lucas that, modulo a prime  $p$ , the binomial coefficients satisfy the congruences

$$\binom{n}{k} \equiv \binom{n_0}{k_0} \binom{n_1}{k_1} \cdots \binom{n_r}{k_r},$$

where  $n_i$ , respectively  $k_i$ , are the  $p$ -adic digits of  $n$  and  $k$ . Many interesting integer sequences have been shown to satisfy versions of these congruences. For instance, Gessel has done so for the numbers used by Apéry in his proof of the irrationality of  $\zeta(3)$ . We make the observation that a sequence satisfies Lucas congruences modulo  $p$  if and only if its values modulo  $p$  can be described by a linear  $p$ -scheme, as introduced by Rowland and Zeilberger, with a single state. This simple observation suggests natural generalizations of the notion of Lucas congruences. To illustrate this point, we derive explicit generalized Lucas congruences for integer sequences that can be represented as certain constant terms. This talk includes joint work with Joel Henningsen.