Schedule of Presentations for VSM PhD Positions
Abstracts see below

Tuesday, January 11, 2021

2.00 pm: **Karimatou Djenabou - P-partition generating function and the quasisymmetric basis ϕ**

2.40 pm: **Thomas Lang - Greedy Learning of Neural Networks with Application to Differential Equations**

3.30 pm: **Alexander Stadler - A determination of the class number of quadratic fields modulo 4**

4.10 pm: **Sampada Kolhatkar - Bivariate Chromatic Polynomials of Mixed Graphs**

Thursday, January 13, 2021

1.00 pm: **Carol Badre - All closed orientable 3-manifolds are central in trisections of 5-manifolds**

1.40 pm: **Hailun Lu - A Strong Law of Large Numbers for Random Set Partial Sum Processes indexed by Sets**

2.30 pm: **Noah Giessing - Creative ways of counting from 0 to n**

3.10 pm: **Frieder Simon - Dynamical properties of predictive coding networks**

Abstracts

Karimatou Djenabou - P-partition generating function and the quasisymmetric basis ϕ (Tuesday, January 11, 2.00 pm)

In discrete mathematics, graphs are one of the structures that have been intensively studied. Those studies have in particular helped to characterize certain properties of graphs and compute some graph invariants. In that direction, (quasi)symmetric functions and graph polynomials such as the chromatic symmetric function are of paramount importance in the study of graphs. My master thesis was concerned with acyclic directed graphs (directed trees). A directed tree can be viewed as a poset P. To the latter, one can associate a P-partition generating function which is a quasisymmetric function. In this talk, we present two expansions of this function in the type-2 quasisymmetric power sums basis ϕ and the leading coefficient of some types of posets.

Thomas Lang - Greedy Learning of Neural Networks with Application to Differential Equations (Tuesday, January 11, 2.40 pm)

We address the Axon Algorithm, a new neural network training method introduced by D. Fokina and I. Oseledets, and its applications. It is a greedy, iterative learning technique that sequentially improves the network's approximation quality by expanding its architecture. After the Axon Algorithm is expounded on, we apply it to the framework of differential equations and report computational results.

Alexander Stadler - A determination of the class number of quadratic fields modulo 4 (Tuesday, January 11, 3.30 pm)

This thesis is concerned with the class numbers of quadratic number fields. These are the field extensions Q(sqrt(d)) of Q where d is in Z. We shortly discuss the class group Cl and proceed with a twofold approach to class numbers: On the one hand, we prove an analytic formula, via Dirichlet series. On the other hand, we use the character group of the class group Cl to study the group Cl/Cl^2, which will lead to an explicit determination of the class numbers modulo 4.

Sampada Kolhatkar - Bivariate Chromatic Polynomials of Mixed Graphs (Tuesday, January 11, 4.10 pm)

For a graph $G=(V,E)$, the chromatic polynomial $\chi_G(x)$ counts the number of vertex colourings as a function of number of colours. Stanley's reciprocity theorem connects the chromatic polynomial with the enumeration of acyclic orientations of the graph G. One way to prove the reciprocity result is via the decomposition of chromatic polynomials as the sum of order polynomials over all acyclic orientations.

The bivariate chromatic polynomial $\chi_G(x, y)$ of a graph $G = (V, E)$, introduced by Dohmen–Pönitz–Tittmann (2003), counts all x-colourings of G such that adjacent vertices get different colours if they are $\leq y$. We extend this notion to mixed graphs, which have both directed and undirected edges. Our main results is a decomposition formula which expresses $\chi_G(x, y)$ as a sum of bivariate order polynomials, and a combinatorial reciprocity theorem for $\chi_G(x, y)$.

Carol Badre - All closed orientable 3-manifolds are central in trisections of 5-manifolds (Thursday, January 13, 1.00 pm)

Gay and Kirby generalised the notion of Heegaard splittings to the trisections of smooth 4-manifolds, which was subsequently generalised to the trisections of piecewise-linear 5-manifolds M by Rubinstein and Tillmann. The construction of Rubinstein and Tillmann takes advantage of a natural colouring on the vertices of particular triangulations of M inherited from a piecewise-linear map $\phi: M \rightarrow \Delta$ where $\Delta $ is a 2-simplex.

An object of interest which is a consequence of this construction is the central submanifold, which is obtained as the preimage $\phi^{-1} (b)$ of the barycentre b of Δ. Each 5-simplex of M contains a 3-cube of the central submanifold. We present a reversal of this process by first constructing a cubing of every closed orientable 3-manifold as a branched cover of S^{3} over the Borromean rings and show that they are central submanifolds of trisections of piecewise-linear 5-manifolds.

This is joint work with Stephan Tillmann.

Hailun Lu - A Strong Law of Large Numbers for Random Set Partial Sum Processes indexed by Sets (Thursday, January 13, 1.40 pm)

The random set is a generalization of the random variable and takes sets as its values. Let A be a Borel measurable subset of [0, ∞)d and let {Xj , j ∈ Nd} be a family of random sets. S(A) is defined as the summation of the random sets with indexes belonging to A. We study a problem of the strong law of large numbers (SLLN) for random set partial sum processes indexed by sets, which gives a deeper insight into how the SLLN behaves. We establish the set-indexed SLLN for random compact sets with the convergence induced by the Hausdorff metric and extend the set-indexed SLLN to random closed sets with respect to Mosco convergence and Wijsman convergence.

Noah Giessing - Creative ways of counting from 0 to n (Thursday, January 13, 2.30 pm)

In this presentation I will talk about the Lefschetz fixed point theorem, an important theorem in the area of equivariant K-theory. We will see how to obtain from it a neat, if weird, way of counting lattice points in polytopes, but also interesting connections to representation theory and the Grothendieck-Riemann-Roch theorem.

Frieder Simon - Dynamical properties of predictive coding networks (Thursday, January 13, 3.10 pm)

Predictive coding networks are a neuroscience-inspired alternative to standard fully-connected networks that recently have gained popularity. We highlight why these networks are more biologically faithful than the standard fully-connected ones, present a dynamical systems formulation of them and using this formalism illustrate some intriguing mathematical properties.