Programme on

"Optimal Transport"

April 15 – June 14, 2019

organized by

Mathias Beiglböck (U Vienna), Alessio Figalli (ETH Zürich), Jan Maas (IST Austria), Robert McCann (U Toronto), Justin Solomon (MIT, Cambridge)

- **Thursday, May 2, 2019**

  15:45 – 16:45 **Florentine Fleißner (TU Munich)**

  New results on the minimizing movement scheme for gradient flows

  Abstract: The variational notion of minimizing movements was introduced by Ennio De Giorgi in his paper *New problems on minimizing movements*, who got his inspiration from a paper by Almgren, Taylor and Wang (both 1993). The minimizing movement scheme for the gradient flow driven by a functional $\psi : X \to (-\infty, +\infty]$ on a metric space (X, d) is given by

  $$u_\tau^n \in \operatorname{argmin}_{v \in X} \left\{ \psi(v) + \frac{1}{2\tau} d^2(v, u_\tau^{n-1} \right\} \quad n \in \mathbb{N}$$

  with time steps $\tau > 0$. It has been part of many works that, under suitable natural coercivity assumptions, the limit curves of the scheme as the time step goes to 0 are solutions to the corresponding gradient flow.
  In this talk, new abstract results on the interrelation between the minimizing movement scheme for gradient flows along a sequence of Γ-converging functionals and the gradient flow driven by the Γ-limit functional are presented, in a general metric space (X, d), establishing a direct connection between the gradient flow motion along the sequence and the gradient flow motion of its limit functional. We are able to allow a relaxed form of minimization in each step of the scheme, and so new relaxation results are presented too.
  The second part of the talk is concerned with the reverse approximation of gradient flows as minimizing movements. We prove that, if X is a finite dimensional Euclidean space and ψ a continuously differentiable function satisfying a quadratic lower bound, then for *every* solution u to the gradient flow (which may have an infinite number of solutions) there exist perturbations $\psi_\tau : X \to \mathbb{R}$ converging to ψ in the Lipschitz norm such that u can be approximated by the minimizing movement scheme along $(\psi_\tau)_{\tau>0}$. This result solves a question raised by Ennio De Giorgi and can be partially extended to infinite dimensional Hilbert spaces. In collaboration with Giuseppe Savaré.

All talks take place at ESI, Boltzmann Lecture Hall!