Vorträge
Tenure Track „DATA DRIVEN PARTIAL DIFFERENTIAL EQUATIONS“

Montag, 25. März 2019, Hörsaal 11

James Nichols
(LJLL Sorbonne Université, Paris)

15:00 Uhr – 15:20 Uhr: Didaktischer Vortrag

„An introduction to data assimilation“

PDE models of physical systems are increasingly used in a variety of scientific settings. Modern PDE models typically require significant computation time to solve, and depend on many (possibly infinitely many) parameters. A common setting is that we have a collection of measurements of the real-world physical system, and we wish to match our PDE model to these observations, or find the underlying parameter values that give the best fitting PDE solution. This is a difficult task, especially when the parameter space is high dimensional.
In this lesson we present a mathematical treatment of the measurement and state estimation problem. With the setting formally stated, we discuss the challenges typically faced, and methods to approximate the PDE to help solve this problem. We will derive one linear state estimation procedure, and briefly discuss a variety of other approaches.

15:50 Uhr – 16:35 Uhr: Wissenschaftlicher Vortrag

„Non-linear reduced modelling and state-estimation for parametric PDEs“

Reduced (or surrogate) modelling replaces a high-resolution and high-dimensional parametric PDE model with some approximation that only loses some known accuracy. Reduced models are designed to allow for fast or closed-form computation for a variety of uncertainty quantification and inverse problems. They are useful for estimating the state of a PDE given some finite set of linear measurements of a real-world system. I will present recent work in reduced models, including a discussion about measurement-adaptive construction of reduced models, worst-case versus average-case error optimization, some results in optimal linear approximations, and experiments in non-linear models.

Dienstag, 26. März 2019, Seminarraum 11

Vladimir Kazeev
(Stanford University)

9:00 Uhr – 9:20 Uhr: Didaktischer Vortrag

„From Gaussian elimination to the adaptive low-rank approximation of quantities of interest in PDE models“

Gaussian elimination is one of the most fundamental techniques for solving systems of linear algebraic equations. In terms of matrix transformations, r steps of forward substitution produce an r-step incomplete pivoted LU decomposition and a rank-r approximation of a given matrix. This allows to reinterpret Gaussian elimination as a method of adaptive cross approximation, which seeks to approximate a given matrix on the basis of r rows and r columns only, without accessing all of its entries. In the context of a PDE with several parameters, cross approximation yields a method for the adaptive sampling and low-rank approximation of a quantity of interest as a function of the parameters. We discuss available quasi-optimality conditions for the resulting approximations and consider heuristic versions of the approach.

9:50 Uhr – 10:35: Wissenschaftlicher Vortrag

„Low-rank tensor structure for the data-driven adaptive solution of PDEs: analysis and algorithms“

The numerical solution of PDEs in finite-precision arithmetic involves discretization, which trades off accuracy against computational tractability. Improving the efficiency of discretization by reducing complexity without compromising accuracy is crucial for a broad range of applications and computational facilities, from supercomputing clusters to embedded systems. The most generic discretizations are based on finite-element spaces that are blind to the intrinsic structure of the PDE and data at hand since they do not exploit it in the course of refinement. However, many PDEs possess special features (such as high dimensionality, singularities, multiple scales and uncertainty) that render generic discretizations inefficient or unfeasible. This often can be addressed by problem-specific methods that effectively hard-code the structure of discrete approximations. In this talk, we consider an alternative, data-driven approach based on the multilevel low-rank structure of functions. Without prescribing any specific approximation spaces, this leaves to an algorithm sufficient freedom for adapting computation not only to the analytical properties of the problem class but also to the data of a particular problem instance.

As a model problem, we consider boundary-value problems for second-order elliptic equations with singularities or high-frequency oscillations. These features may require very fine discretizations to allow for accurate numerical simulation. Instead of using finite-element bases adapted to the respective problem classes, one can retain generic discretizations, based on uniform meshes and tensor-product finite-element spaces in the physical or in a reference domain, but parametrize them by multidimensional arrays with low-rank tensor structure. We consider the structure known as matrix product states (MPS) in computational quantum chemistry and as tensor train (TT) in numerical analysis. The TT/MPS structure is remarkable for its close connection with low-rank matrix factorizations, affording robust and efficient computation within the format. This allows to never access the vast generic finite-element spaces underlying the discretization directly and to work exclusively with their nonlinear low-rank parametrizations instead. With a rigorous analysis and algorithmic treatment of several new phenomena and challenges related to the arithmetic, numerical stability and conditioning of such low-rank tensor factorizations, we achieve optimal convergence using fine meshes with width at the level of machine precision.
