PhD Colloquium

June 7th 2017, 15:15–16:00

HS 05

Cardinal Invariants of the Continuum and the Structure of the Real Line

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After Cantor's revolutionary discovery regarding the cardinality of the set of real numbers which states that the size of the set of real numbers is not countable, the first cardinal invariant we know appeared; it is precisely the cardinality of the set of reals $\mathfrak{c} = |\mathbb{R}|$, or as many call it, the *cardinality of the continuum*. One main question that arose after Cantor's work in real analysis, for example, was whether or not some properties of the real line that were known to be valid for countably many subsets of the reals could be extended to $\mathfrak{c}$-many such sets.
Cardinal invariants of the continuum are cardinals describing mostly the combinatorial or topological structure of the real line. In simple words, they are simply the smallest cardinals $\leq \mathfrak{c}$ for which various results, true for $\aleph_0$, become false. They are usually defined in terms of ideals on the reals, or some very closely related structure such as $\mathcal{P}(\omega)/\operatorname{fin}$ and typically they assume values between $\aleph_1$, the first uncountable cardinal and $\mathfrak{c}$. Hence, they are uninteresting in models where CH holds. However, in models of set theory where the continuum hypothesis fails they may assume different values and interact with each other in several ways.
In this talk, I will present the basics of the subject, some interesting problems and show some of the usual techniques used in set theory to decide the values of these cardinals. Finally, I will talk about some of the current lines of research and mention some open problems.

Master Students, PhD Students, Postdocs and other members of the Faculty of Mathematics are cordially invited!

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