Labyrinth fractals and their magic

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Abstract

An $n \times n$ *pattern* is obtained by starting with the unit square, dividing it into $n \times n$ congruent smaller subsquares and colouring some of them in black (which means that they will be cut out), and the rest in white.

Sierpiński carpets are (self-similar) fractals in the plane that originate from the well-known Sierpiński carpet. They are constructed in the following way: one starts with the unit square, divides it into $n \times n$ congruent smaller subsquares and cuts out m of them, corresponding to a given $n \times n$ pattern (also called the generator of the Sierpiński carpet). This construction step is then repeated with all the remaining subsquares ad infinitum. The resulting object is a fractal of Hausdorff and box-counting dimension $\log(n^2 - m)/\log(n)$, called a *Sierpiński carpet*.

By using special patterns, which we called "labyrinth patterns", we created and studied a special class of carpets, called labyrinth fractals. Labyrinth fractals are self-similar and under certain conditions on the patterns one obtains objects with some "magic" properties. Later on we introduced and studied mixed labyrinth fractals, that are not self-similar. Also other generalisations are possible.

During this talk we will also see how, by choosing the labyrinth patterns in an appropriate way, one can obtain... almost anything.

The results stem from joint work with Bertran Steinsky and Gunther Leobacher.