Workshop on

"Geometry and Representation Theory"

January 16 – 27, 2017

organized by

Tomoyuki Arakawa (RIMS, U of Kyoto), Karin Baur (U of Graz), Victor Kac (MIT, Boston), Anne Moreau (U of Poitiers)

Week 1

January 16 – 20, 2017

- **Monday, January 16, 2017**

  08:30 – 09:00 **Opening & Registration**

  09:00 – 10:00 **Pramod Achar**
  Title: Introduction to affine Grassmannians and the geometric Satake equivalence I.
  Abstract: The goal of these lectures will be to start from scratch and learn what the affine Grassmannian of a reductive group is, and how its geometry is related to the representation theory of the Langlands dual group. I will emphasize the examples of GL_n and SL_2 throughout. Here is an outline of the three lectures:

  1. Lecture 1: definitions; lattice model for GL_n; orbits.

  2. Lecture 2: convolution product; Lusztig's q-analogue of the weight multiplicity.

  3. Lecture 3: geometric Satake equivalence; MV cycles.

  10:05 – 11:05 **Guido Pezzini**
  Title: Geometry and combinatorics of spherical varieties I.
  Abstract: In these lectures we will introduce spherical varieties and discuss some of their basic properties; we will also introduce related combinatorial objects and see how they govern the geometry of such varieties. The topics will be:

  1. Definitions, examples, first properties.

  2. Local structure theorem.

  3. The Luna-Vust theory of embeddings.

  4. Spherical roots and the multiplication of regular functions.

  11:05 – 11:35 *coffee / tea break*

  11:35 – 12:35 **Ivan Mirkovic**
  Title: Generalized affine Grassmannians I.
  Abstract:

  12:35 – 14:30 *lunch break*

  14:30 – 15:30 **Pavel Etingof**
  Title:I Introduction to D-modules I.
  Abstract: I will give a brief introduction to the theory of algebraic D-modules, with an emphasis on the

notions to be used in D. Gaitsgory's minicourse. I will review the following topics (time permitting):

1. Definition of the category of D-modules on a smooth variety.

2. Equivalence between left and right D-modules.

3. Kashiwara's theorem and D-modules on singular varieties.

4. *-direct and !-inverse images, adjunctions between them for open embeddings and proper morphisms, base change.

5. Singular support, Gabber's theorem.

6. Holonomic D-modules.

7. Verdier duality.

8. Formalism of six functors.

9. Beilinson-Bernstein localization theorem.

15:30 – 15:50 *break*

15:50 – 16:45 **Pierre Baumann**
Title: Bases for tensor products and geometric Satake correspondence.
Abstract: Let G be a connected reductive group over $\mathbb{C}$. The geometric Satake correspondence, due to Lusztig, Ginzburg, Beilinson-Drinfeld, and Mirković-Vilonen, asserts that the Langlands dual $G^\vee$ of G can be obtained by Tannakian reconstruction from a category of perverse sheaves on the affine Grassmannian of G. Specifically, tensor products of simple representations of $G^\vee$ are obtained as the intersection homology of convolution varieties. One may then define bases of these tensor products, indexed by algebraic cycles defined by Braverman-Gaitsgory, Gaussent-Littelmann-Nguyen, and Goncharov-Shen. We will explain that these bases share many properties with Lusztig's dual canonical bases, thereby generalizing previous results by Fontaine-Kamnitzer-Kuperberg. This is joint work with S. Gaussent and P. Littelmann.

- **Tuesday, January 17, 2017**

  09:00 – 10:00 **David Hernandez**
  Title: R-matrices, affine quantum groups and applications I.
  Abstract: R-matrices are solutions of the quantum Yang-Baxter equation. At the origin of the theory of quantum groups, they can be interpreted as intertwining operators in representation theory. After reviewing standard constructions from quantum affine algebras, we will present recent development in the theory. Maulik-Okounkov gave a general geometric construction of stable basis and R-matrices. In another direction, monoidal categorifications of cluster algebras have been established using R-matrices to categorify Fomin-Zelevinsky mutations relations. If time allows, we will also discuss recent advances on transfer matrices derived from R-matrices, which give new informations on corresponding quantum integrable systems as well as on the ODE/IM correspondence seen in the context of affine opers.
  Here is a tentative plan for the lectures :

  1. R-matrices, algebraic and geometric constructions.

  2. Categorification of mutation relations.

  3. Transfer matrices and spectra of quantum integrable models.

  4. Langlands duality and affine opers.

  10:05 – 11:05 **Pavel Etingof**
  Title: Introduction to D-modules II.
  Abstract: see lecture I.

  11:05 – 11:35 *coffee / tea break*

  11:35 – 12:35 **Pavel Etingof**
  Title: Introduction to D-modules III.

Abstract: see lecture I.

12:35 – 14:30 *lunch break*

14:30 – 15:30 **Guido Pezzini**
Title: Geometry and combinatorics of spherical varieties II.
Abstract: see lecture I.

15:30 – 15:50 *break*

15:50 – 16:45 **Maria Gorelik**
Title: Duflo-Serganova functor for affine Lie superalgebras.
Abstract: The Duflo-Serganova functor introduced by Dulfo and Serganova in arXiv:math/0507198. assigns to a Lie superalgebra $\mathfrak{g}$ another Lie superalgebra $\mathfrak{g}_x$ of a smaller defect and provides a functor from the $\mathfrak{g}$-modules to the $\mathfrak{g}_x$-modules. In my talk I will present some results on Duflo-Serganova functor for affine Lie superalgebras. This is a joint work in progress with Vera Serganova.

- **Wednesday, January 18, 2017**

  09:00 – 10:00 **Pramod Achar**
  Title: Introduction to affine Grassmannians and the geometric Satake equivalence II.
  Abstract: see lecture I.

  10:05 – 11:05 **David Hernandez**
  Title: R-matrices, affine quantum groups and applications II.
  Abstract: see lecture I.

  11:05 – 11:35 *coffee / tea break*

  11:35 – 12:35 **Dennis Gaitsgory**
  Title: Chiral algebras I.
  Abstract:

  1. Local theory of chiral algebras.
     (a) Definition
     (b) Commutative chiral algebras
     (c) Lie-* algebras and chiral universal envelopes
     (d) Basic examples: Kac-Moody, Heisenberg.
     (e) Example: chiral algebra of differential operators
  2. Factorization approach and chiral homology
     (a) Ran space
     (b) Factorization algebras and equivalence with chiral algebras
     (c) Chiral homology
     (d) Sample calculations of chiral homology: the commutative case, univ. env. case.
  3. Applications to the geometric Langlands theory:
     (a) Localization of Kac-Moody representations on Bun_G.
     (b) The case of the chiral algebra of differential operators.
     (c) The integrable quotient of the Kac-Moody chiral algebra.
     (d) The chiral algebra responsible for the quantum Langlands correspondence.

  12:35 – 14:30 *lunch break*

  14:30 – 15:30 **Short talks by students: Carpentier** (*Integrable operators*), **Fujita** (*Tilting modules in affine highest weight categories*), **Jensen** (*The p-canonical basis of Hecke algebras and p-cells*), **Kusumastuti** (*Kostant principal filtration and paths in weight lattices*), **Ruotao** (*Another definition of singular support for D modules*).

15:30 – 15:45 *break*

15:45 – 16:45 **Short talks by students: Bittmann** (*Baxter relations and quantum Grothendieck rings*), **Allegra** (TBA), **Przezdziecki** (*The Suzuki functor at the critical level*), **Patimo** (*The Neron-Severi Lie algebra of a Schubert Variety*), **Hebert** (*Gindikin-Karpelevich finiteness for Kac-Moody groups over local fields*).

- **Thursday, January 19, 2017**

  09:00 – 10:00 **Dennis Gaitsgory**
  Title: Chiral algebras II.
  Abstract: see lecture I.

  10:05 – 11:05 **Victor Batyrev**
  Title: Spherical varieties and arc spaces I.
  Abstract: Arc spaces of algebraic varieties turn out to be very useful for defining some topological invariants of algebraic varieties via a motivic integration on their smooth birational models. The arc spaces of spherical homogeneous spaces have been used implicitly in the Luna-Vust theory (1983) of equivariant valuations and spherical embeddings. The purpose of my talks is to explain my recent joint results with Anne Moreau on arc spaces of spherical embeddings. These results allow in particular to derive a combinatorial formula for computing Betti numbers of an arbitrary smooth projective spherical variety.

  11:05 – 11:35 *coffee / tea break*

  11:35 – 12:35 **David Hernandez**
  Title: R-matrices, affine quantum groups and applications III.
  Abstract: see lecture I.

  12:35 – 14:30 *lunch break*

  14:30 – 15:30 **Ivan Mirkovic**
  Title: Generalized affine Grassmannians II.
  Abstract: see lecture I.

  15:30 – 15:50 *break*

  15:50 – 16:45 **Alexander Premet**
  Title: Classification of the maximal subalgebras of exceptional Lie algebras in good characteristic
  Abstract: Let G be an exceptional simple algebraic group over an algebraically closed field of characteristic $p > 3$. If G is of type E_8 suppose that $p > 5$. In my talk based on a joint work with David Stewart I will describe the maximal Lie subalgebras of the Lie algebra of G.

- **Friday, January 20, 2017**

  09:00 – 10:00 **Victor Batyrev**
  Title: Spherical varieties and arc spaces II.
  Abstract: see lecture I.

  10:05 – 11:05 **Pramod Achar**
  Title: Introduction to affine Grassmannians and the geometric Satake equivalence III.
  Abstract: see lecture I.

  11:05 – 11:35 *coffee / tea break*

  11:35 – 12:35 **Ivan Mirkovic**
  Title: Generalized affine Grassmannians III.
  Abstract: see lecture I.

  12:35 – 14:30 *lunch break*

14:30 – 15:30 **Dennis Gaitsgory**
Title: Chiral algebras III.
Abstract: see lecture I.

15:30 – 15:50 *break*

15:50 – 16:45 **Daniel Juteau**
Title: Support of the spherical representation of the rational Cherednik algebra.
Abstract: I will give a very simple criterion, obtained in collaboration with Stephen Griffeth, to determine the support of the spherical representation of a general rational Cherednik algebra (associated to any complex reflection group, and any parameters), assuming some conjectures on Hecke algebras which are known for Coxeter groups but not for all complex reflection groups. In any case, we obtain a much simpler proof of Etingof's criterion for Coxeter groups (which itself generalized a result of Varagnolo and Vasserot for Weyl groups with equal parameters), and a simpler criterion for the infinite series than what was previously known.

All talks take place at the ESI, Boltzmann Lecture Hall!