

S -adic dynamical systems

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Let S be a finite set of substitutions over a given finite alphabet \mathcal{A} and choose a sequence $(\sigma_n)_{n \in \mathbf{N}} \in S^{\mathbf{N}}$. For some sequence $i_n \in \mathcal{A}$ define an infinite word $w \in \mathcal{A}^{\mathbf{N}}$ by

$$w = \lim_{n \rightarrow \infty} \sigma_1 \circ \cdots \circ \sigma_n(i_n i_n \dots).$$

If Σ denotes the shift on $\mathcal{A}^{\mathbf{N}}$ we study dynamical systems of the form (X_w, Σ) where

$$X_w = \overline{\{\Sigma^k w \mid k \in \mathbf{N}\}}.$$

We relate these dynamical systems to generalized continued fraction algorithms, show that they are conjugate to rotations on a torus under certain conditions, and use them to define nonstationary Markov partitions for “mapping sequences” of toral automorphisms.