THE STRING DENSITY PROBLEM AND THE CAMASSA–HOLM EQUATION

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The following ODE (known as *the string spectral problem*)

$$-y'' = z\,\omega\,y, \quad x \in [0, L), \tag{1}$$

is a classical object in spectral theory. Here $L \in (0, \infty]$ is the length of the string, ω is a positive Borel measure on $[0, L)$ called the mass density of the string, and z is a spectral parameter. Recently, the string spectral problem has come up in connection with some completely integrable nonlinear wave equations (e.g., the Camassa–Holm equation) for which the string spectral problem serves as an underlying isospectral problem. In contrast to the KdV equation, the Camassa–Holm equation possesses peaked solitons, called peakons, and models breaking waves. The latter happens when ω is a signed measure, i.e., the string is *indefinite*.

In this talk, we review the direct and inverse spectral theory for indefinite strings and relate it to the conservative Camassa–Holm flow. As one of our main results we are going to present the indefinite analog of M. G. Krein's celebrated solution to the string density problem.

The talk is based on joint work with Jonathan Eckhardt.

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