universität wien

Fakultät für Mathematik

Mathematisches Kolloquium und Junior Kolloquium

Hedy Attouch

(Université Montpellier 2, France)

15:00 Uhr

JUNIORKOLLOQUIUM

“Gradient-based methods for Multiobjective Optimization.
A dynamical approach to Pareto optima.”

Our goal is to present gradient-based dynamics and algorithms for multiobjective optimization. The following model works in a general Hilbert space, which makes it applicable to a wide variety of problems in decision sciences, and inverse problems in engineering. For simplicity of presentation, consider a finite family of differentiable objective functions $\{f_i;\ i = 1,2,...q\}$, and the unconstrained case. The vector field which governs the dynamic is

$$s(x) := -(\mathrm{Conv}\{\nabla f_i(x)\})^0$$

where $\mathrm{Conv}\{\nabla f_i(x)\}$ is the convex hull of the finite set of gradient vectors $\{\nabla f_i(x);\ i = 1,2,...q\}$ at x (a polyhedral convex set), and $(\mathrm{Conv}\{\nabla f_i(x)\})^0$ is its element of minimal norm. The associated dynamical system is called the Multi-Objective Gradient system ((MOG) for short)

$$(MOG)\ \dot{x}(\mathrm{t}) + (\mathrm{Conv}\{\nabla f_i(x)\})^0 = 0.$$

This dynamic system has been introduced by economists (Smale, Henry, Cornet) in the 80th for the optimal allocation of resources. It is only recently that its fundamental importance in the optimization was highlighted, see [1] and references therein.

We first present the properties of (MOG) that make it attractive in terms of modeling:

- It is a descent method, i.e., for *each* $i = 1,...,q$, t 7→ $f_i(x(t))$ is nonincreasing.
- Its trajectories converge to weak Pareto optimal points when the f_i are convex, and critical Pareto points when the f_i are quasi-convex.
- At time t, the vector field which governs (MOG) is a convex combination $\sum_{i=1}^{q} \theta_i(t) \nabla f_i(\cdot)$ of the gradients with scalars which are not fixed in advance. They are part of the process.

Then, we show how to extend the dynamical system to the constrained case, and non-smooth objective functions.

Discretization of the dynamical system provide algorithms which share the same properties, which makes the link with the multiobjective steepest descent algorithm by Fliege and Svaiter [2], and the associated splitting methods.

Finally, we present recent results concerning the inertial version of (MOG) (second-order differential equation), some open problems, and directions of research.

16:15 Uhr

VORTRAG

“FAST SPLITTING ALGORITHMS FOR CONVEX OPTIMIZATION.
BEYOND NESTEROV COMPLEXITY BOUND $O(1/k^2)$”.

Many scientifc and engineering problems can be naturally modeled as very large scale optimization problems, creating new challenges for the optimization discipline. In this perspective, in recent years, considerable effort has been devoted to the study of first-order splitting algorithms. The *forward-backward algorithm*, which is one of the most important, is a powerful tool for solving optimization problems with a *additively separable* and *smooth* plus *nonsmooth* structure. In the convex setting, a simple but ingenious acceleration scheme developed by Nesterov improves the theoretical rate of convergence for the function values from the standard $O(k^{-1})$ down to $O(k^{-2})$. In this lecture, we show that the rate of convergence of a slight variant of Nesterov's accelerated forward-backward method, which produces *convergent* sequences, is actually $o(k^{-2})$, rather than $O(k^{-2})$. Our arguments are based on the connection between this algorithm and a second-order differential inclusion with vanishing damping, recently introduced by Su, Boyd and Candès. The key point is the introduction of energy-like Lyapunov functions, with adapted scaling. Linking algorithms with dynamical systems provide connections between different areas, and a valuable guide for the proofs. Finally, we consider the hierarchical multi-objective problem which consists in finding by rapid methods the solution with minimum norm of a convex minimization problem. To this end, we introduce into the dynamics and algorithms a Tikhonov regularization term with vanishing coefficient. Applications are given in sparse optimization for signal/imaging processing, and inverse problems. We conclude by showing some recent directions of research, in particular the developments of these methods to nonconvex nonsmooth semi-algebraic problems, based on Kurdyka-Lojasiewicz inequality.

Mittwoch, 11. Mai 2016
15.00 Uhr Juniorkolloquium
15.45 Uhr – 16.15 Uhr Kaffeejause
16.15 Uhr Vortrag

Vinum cum pane im Anschluss

Ort: Fakultät für Mathematik,
Oskar-Morgenstern-Platz 1, Sky Lounge

Radu Bot
Harald Rindler