DVR 0065528

Programme on

"Mixing Flows and Averaging Methods"

April 4 – May 25, 2016

organized by

Peter Bálint (TU, Budapest), Henk Bruin (U Vienna), Carlangelo Liverani (U Rome, Tor Vergata), Ian Melbourne (U Warwick), Dalia Terhesiu (U Exeter)

Workshop 1

"Thermodynamic Formalism and Mixing"

April 25 – 29, 2016

- **Monday, April 25, 2016**

  10:00 – 10:30 **Opening & Registration**

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Frédéric Faure**
  Global normal form and asymptotic spectral gap for open partially expanding maps

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Masato Tsujii**
  Exponential mixing for generic volume-preserving Anosov flows in dimension three

  15:00 – 15:30 *break*

  15:30 – 16:30 **Benoît Saussol**
  Linear response in the intermittent family

- **Tuesday, April 26, 2016**

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Mark Holland**
  Almost sure convergence of maxima for chaotic dynamical systems

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Masato Tsujii**
  Exponential mixing for generic volume-preserving Anosov flows in dimension three

  15:00 – 15:30 *break*

  15:30 – 16:30 **Vaughn Climenhaga**
  Equilibrium states for geodesic flow in nonpositive curvature

- **Wednesday, April 27, 2016**

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Wael Bahsoun**
  Mixing rates and limit theorem for random intermittent maps

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Masato Tsujii**
  Exponential mixing for generic volume-preserving Anosov flows in dimension three

  15:00 – 15:30 *break*

  15:30 – 16:30 **Evgeny Verbitzkiy**
  Random Continued Fraction Expansions

- **Thursday, April 28, 2016**

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Tom Kempton**
  Ergodic Theory of the Scenery Flow and Self-Affine Sets

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Mike Todd**
  Continuity in thermodynamic formalism

  15:00 – 15:30 *break*

  15:30 – 16:30 **Paulo Varandas**
  Contributions to the thermodynamic formalism of semigroup actions

- **Friday, April 29, 2016**

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Michael Jakobson**
  Ergodic properties of some attractors with countable Markov partitions

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Godo Iommi**
  Phase transitions for suspension and geodesic flows

  15:00 – 15:30 *break*

  15:30 – 16:30 **Peyman Eslami**
  Coupling for piecewise expanding maps

All talks take place at the ESI, Boltzmann Lecture Hall!

Titles and abstracts

- Wael Bahsoun "Mixing rates and limit theorem for random intermittent maps."
  Abstract: In this talk we present recent results on the statistical properties for random intermittent maps that share a common neutral fixed point. We illustrate how the constituent map that is fastest mixing dominates the asymptotic properties of the random map. We establish sharp estimates on the position of return time intervals for the quenched dynamics of the random system. This allows us to prove annealed limit theorems (CLT, stables laws) in the probabilistic case, and to obtain correlation asymptotics in the infinite measure preserving case. This is a joint work with Chris Bose (Victoria, Canada).

- Vaughn Climenhaga "Equilibrium states for geodesic flow in nonpositive curvature"
  Abstract: The geodesic flow on a negatively curved manifold is one of the classical examples of a uniformly hyperbolic (transitive Anosov) system; in particular, it has a unique measure of maximal entropy, and more generally, unique equilibrium states for Holder continuous potentials. When curvature is only assumed to be non-positive, the geodesic flow becomes non-uniformly hyperbolic and much less is known. For a rank 1 manifold of non-positive curvature, Knieper showed uniqueness of the measure of maximal entropy, but his methods do not generalize to equilibrium states for non-zero potentials.
  I will discuss joint work with Keith Burns, Todd Fisher, and Daniel J. Thompson, in which we use a non-uniform version of Bowens specification property to establish existence and uniqueness of equilibrium states for a class of non-zero potential functions. This class includes scalar multiples of the geometric potential for an interval of parameter values. Our methods also have applications to partially hyperbolic diffeomorphisms.

- Peyman Eslami "Coupling for piecewise expanding maps"
  Abstract: I will explain the coupling method for 1D piecewise expanding maps of the interval. This is a method to obtain statistical properties of dynamical systems in particular decay of correlations. In comparison to other methods it is more flexible and leads to explicit constants. In the past it has been applied in more complicated systems e.g. Billiards but such systems also have more structure. Here we focus on 1D piecewise expanding maps, but our goal is to devise a method that is flexible enough to be generalized to higher dimensional systems.

- Frédéric Faure "Global normal form and asymptotic spectral gap for open partially expanding maps"
  Abstract: Joint work with Tobias Weich. We are interested in the quantity $\gamma_{asympt.} := \limsup_{\omega\to 0} \log\left(r_s\left(\mathcal{L}_\omega\right)\right)$, namely the logarithm of the spectral radius of the transfer operator $\mathcal{L}_\omega u := e^{i\omega\tau+V} u \circ E$ in the limit of high frequencies ω, where τ, V are smooth functions and E is an expanding map on intervals. Under some hypothesis it is known from D. Dolgopyat (2002) that $\exists \epsilon > 0, \gamma_{asympt.} \leq \gamma_{Gibbs} - \epsilon$ with $\gamma_{Gibbs} = \Pr\left(V - \log|E'|\right)$ and $\Pr(.)$ being the topological pressure. Using semiclassical analysis it is known that $\gamma_{asympt.} \leq \gamma_{sc} = \sup\left(V - \frac{1}{2}\log|E'|\right)$. We show that $\gamma_{asympt} \leq \gamma_{up} := \frac{1}{2}\Pr\left(2\left(V - \log|E'|\right)\right) + \frac{1}{4}\left\langle \log|E'|\right\rangle$ where $\langle . \rangle$ is an "average" and we will discuss the conjecture that generically $\gamma_{asympt} = \frac{1}{2}\Pr\left(2\left(V - \log|E'|\right)\right)$.

- Mark Holland "Almost sure convergence of maxima for chaotic dynamical systems"
  Abstract: Suppose $(f, \mathcal{X}, \nu)$ is a measure preserving dynamical system and $\phi : \mathcal{X} \to \mathbb{R}$ is an observable with some degree of regularity. We investigate the maximum process $M_n := \max(X_1, \ldots, X_n)$, where $X_i = \phi \circ f^i$ is a time series of observations on the system. When $M_n \to \infty$ almost surely, we establish results on the almost sure growth rate, namely the existence (or otherwise) of a sequence $u_n \to \infty$ such that $M_n/u_n \to 1$ almost surely. For a wide class of non-uniformly hyperbolic dynamical systems we determine where such an almost sure limit exists and give examples where it does not.

- Godo Iommi "Phase transitions for suspension and geodesic flows"
  Abstract: In this talk I will first describe joint work with Thomas Jordan in which we studied suspension flows defined over countable Markov shifts. We established conditions for the pressure function to be real analytic or to exhibit a phase transitions. Our results depended on a symbolic parameter of the flow. In recent joint work with Felipe Riquelme and Anibal Velozo, we studied thermodynamic formalism for a class of geodesic flows defined over certain non-compact manifolds. Flows in this class have symbolic representations as suspension flows over countable Markov shifts. I will describe the construction of a class of potentials exhibiting phase transitions and a geometric characterization of the relevant symbolic parameter.

- Michael Jakobson "Ergodic properties of some attractors with countable Markov partitions"
  Abstract: We study certain piecewise smooth two-dimensional systems with countable Markov partitions. In particular we prove exponential decay of correlations by using several results of O. Sarig. Our approach is motivated by the original method of Anosov and Sinai from their 1967 paper.
- Tom Kempton "Ergodic Theory of the Scenery Flow and Self-Affine Sets"
  Abstract: The scenery flow describes the process of zooming in on a set or measure about a point. It has had substantial recent success in resolving old questions in fractal geometry and geometric measure theory. For self-similar sets the corresponding scenery flow has a simple dynamical description. In this talk I will give an introduction to the ergodic theory of scenery flows before going on to consider the question of the scenery flow for self-affine sets and measures.
- Benoît Saussol "Linear response in the intermittent family"
  Abstract: We prove a linear response formula for the intermittent family $x \mapsto x + 2^{\alpha}x^{1+\alpha}$, that is the differentiability of the density h_{α} with respect to the parameter α. This is a joint work with Wael Bahsoun.
- Mike Todd "Continuity in thermodynamic formalism"
  Abstract: Ill discuss recent results with Neil Dobbs on the continuity of measures, entropy and pressure for a broad class of interval maps. The conditions for continuity of these quantities are sharp. The approach combines basic continuity ideas with careful estimates on the combinatorics of Markov extensions.
- Paulo Varandas "Contributions to the thermodynamic formalism of semigroup actions"
  Abstract: In this talk we will report on some recent results concerning the topological entropy and dynamics of continuous free semigroup actions. We give sufficient topological conditions for the topological entropy to be positive and to be given by the exponential growth rate of 'non-autonomous' periodic points. In the case of semigroups of Ruelle-expanding maps, parameterized by random walks on the free semigroup, we will discuss the regularity of the pressure function, establish the main properties of the dynamical zeta function and prove the existence of stationary probability measures.
- Masato Tsujii "Exponential mixing for generic volume-preserving Anosov flows in dimension three"
  Abstract: Let M be a closed 3-dimensional Riemann manifold and let $3 \leq r \leq \infty$. We prove that there exists an open dense subset in the space of C^r volume-preserving Anosov flows on M such that all the flows in it are exponentially mixing.
- Evgeny Verbitskiy "Random Continued Fraction Expansions"
  Abstract: We introduce a random dynamical system related to continued fraction expansions. It uses random combination of the Gauss map and the Rényi (or backwards) continued fraction map. We explore the continued fraction expansions that this system produces as well as the dynamical properties of the system. Finally, we will discuss existence and properties of the invariant density. The talk is based on a joint work with C. Kalle, T. Kempton, and M. Tsujii.