Simons Lecture Series

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Schrödinger Lecture Hall, ESI, Boltzmanngasse 9, Vienna

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Operads, factorization algebras, and (topological) quantum field theory with a flavor of higher categories

The main goal of this Lecture Course is to give an introduction to certain algebraic structures arising in the study of (topological) quantum field theory.

Operads, introduced by May, form a framework to abstract families of composable functions in a systematic way and have become appeared in many fields of mathematics, such as topology, homological algebra, algebraic geometry, and mathematical physics. We will encounter algebraic examples such as the associative and commutative operad (encoding associative and commutative algebras) and examples of a topological nature, called the little n-disks operad (encoding the structure of n-fold loop spaces).

Factorization algebras, first introduced by Beilinson and Drinfeld in an algebro-geometric context, are algebraic structures which give a way to encode the structure of observables of a perturbative quantum field theory. Examples include (homotopy) algebras and (pointed) bimodules, but also braided monoidal categories such as the category of finite dimensional representations of a reductive algebraic group $\mathrm{Rep}\, G$ or of the associated quantum group $\mathrm{Rep}\, U_q(g)$. In the situation when the field theory is topological, we essentially get E_n-algebras, which are algebras for the little n-disks operad mentioned above.

A connection of the above concepts to topological field theories is given by factorization homology. We will see how higher category techniques enter the picture and, time permitting, see how this leads to an example of a (fully extended) topological field theory à la Atiyah-Segal. Possible topics for an outlook to ongoing research are so-called twisted field theories in the sense of Stolz-Teichner, which are related to boundary field theories, or factorization homology for higher categories after Ayala-Francis-Rozenblyum..

Nils Carqueville