Exponential asymptotic splitting for the linear Schrödinger equation

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The discretization of a linear Schrödinger equation is difficult due to the presence of a small parameter which induces high oscillations. A standard approach consists of a spectral semidiscretization, followed by an exponential splitting. This, however, is sub-optimal, because the exceedingly high precision in space discretization is married by low order of the time solver.
In this talk we sketch an alternative approach. Our analysis commences not with semi-discretisation, but with the investigation of the free Lie algebra generated by differentiation and by multiplication with the interaction potential: it turns out that this algebra possesses a structure which renders it amenable to a very effective form of *exponential asymptotic splitting*: exponential splitting where consecutive terms are scaled by increasing powers of the small parameter. The semi-discretisation in deferred to the very end of computations.
We will focus on the method for the time dependant linear Schrödinger equation with potential non-depending on time, however we will also discuss the difficulties that appear with time dependant potential and will briefly propose the remedy to that stage of an affair.

This talk is based on a joined work with Philipp Bader (La Trobe University, Australia), Iserles Arieh (University of Cambridge, UK) and Pranav Singh (University of Cambridge, UK)