SOME QUESTIONS RELATED TO ELLIPTIC FRACTIONAL LAPLACIAN EQUATIONS WITH NEUMANN BOUNDARY CONDITIONS

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In this talk we consider the following fractional semilinear Neumann problem on a smooth bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$,

$$\begin{cases} (-\varepsilon\Delta)^{1/2}u + u = u^p, & \text{in } \Omega, \\ \partial_\nu u = 0, & \text{on } \partial\Omega, \\ u > 0, & \text{in } \Omega, \end{cases}$$

where $\varepsilon > 0$ and $1 < p < (n+1)/(n-1)$. This is the fractional version of the semilinear Neumann problem studied by Lin–Ni–Takagi in the late 80's. The problem arises by considering steady states of the Keller–Segel model with nonlocal chemical concentration diffusion. Using the semigroup language for the extension method and variational techniques, we will comment the methods allowing to obtain the existence of nonconstant smooth solutions for small ε. In the case of large ε we give results about nonexistence of nonconstant solutions, and it will be also shown that as $\varepsilon \to 0$ the solutions u_ε tend to zero in measure on Ω, while they form spikes in $\overline{\Omega}$. The regularity estimates of the fractional Neumann Laplacian that we develop are essential for the analysis. These results are based on a joint work with Pablo Raúl Stinga.