Existence and optimal rate of convergence to equilibrium for solutions to the homogeneous Boltzmann equation with a Maxwellian kernel

Emanuele Dolera

Università di Modena e Reggio Emilia, Italy

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Abstract

This talk deals with a well-known equation in the kinetic theory of gases, namely the spatially homogeneous Boltzmann equation. Conceived by the Austrian physicist L.Boltzmann in 1872 even in a more complete form, the equation at issue governs time evolution of the probability distribution of the particle velocities in a dilute gas. Spatial homogeneity is a suitable mathematical simplification of the complete model which highlights the effect of binary collisions on the system, and the specification of "Maxwellian kernel" means we are focusing on a specific class of physical interactions between particles, whose prototype is the purely repulsive force inversely proportional to the fifth power of their distance (Maxwell force). We present three kinds of new results: A probabilistic representation for the solution in terms of a random sum of conditionally independent random variables, an existence result for the Boltzmann equation under the weakest assumption known so far on the collision kernel, and a definitive answer to the problem of quantifying the relaxation to equilibrium for Boltzmann-equation solutions under really mild conditions on the initial datum. This last result, which states that the total variation distance (i.e. the L^1-norm in the absolutely continuous case) between the solution and the limiting Maxwellian distribution admits an upper bound of the form $Ce^{-\Lambda_b^* t}$, Λ_b^* being the spectral gap of the linearized collision operator and C a constant depending only on the initial datum, is at the core of kinetic theory since it provides a detailed indication about the time-scale for relaxation to equilibrium. D.Hilbert hinted at the validity of the above quantification in 1912, which was explicitly formulated as a conjecture by H.P.McKean in 1966. The main line of the new proof is based on an analogy between the problem of convergence to equilibrium and the central limit theorem of probability theory, as suggested by McKean.