The Impact of Smoothness on Inverse Problems and Regularization: a Curse or a Blessing?

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Abstract:

During the last ten years, there has been a significant progress with respect to the development and analysis of approximate solutions to nonlinear inverse problems. This rapid expansion has been caused by requirements of applications that arise in natural sciences, engineering, imaging, and finance. The inverse problems can be written, in general, as operator equations $F(x) = y$ with a nonlinear forward operator F mapping between the Hilbert or Banach spaces X and Y. In this context, $x \in X$ is the non-observable element to be determined from noisy observations of the element $y \in Y$, which can be interpreted as an effect caused by x. Unfortunately, it is an intrinsic property of F to be 'smoothing', which means that during the transition from x to y information gets lost as is typical for procedures including integration. Consequently, the retrieval of x from y tends to become unstable as is typical for procedures including differentiation. This is the phenomenon of ill-posedness. Therefore the stable approximate solution of inverse problems requires regularization based on the substitution of the ill-posed original problem by a well-posed auxiliary problem. For a wide range of regularization methods that have been studied, an error analysis could be developed. In particular, to derive convergence rates for regularization methods some kind of smoothness of the solution is necessary. From this perspective, smoothness is a welcome property occurring in the solution process of inverse problems. This talk presents, in addition to theoretical ingredients, examples of the treatment of nonlinear applied inverse problems from technology, laser optics and from the financial markets. Moreover, some new results on ℓ^1-regularization are shown when the sparsity assumption is narrowly missed.