ESI Erwin Schrödinger International Institute for Mathematical Physics

universität wien

DVR 0065528

Workshop on

"Ergodic Theory and Holomorphic Dynamics"

September 28 – October 2, 2015

organized by

Anna Miriam Benini (Rome II - Tor Vergata), Henk Bruin (U Vienna),
Dierk Schleicher (Jacobs University Bremen), Sebastian van Strien (Imperial College)

- **Monday, September 28, 2015**

  09:00 – 09:30 **Opening & Registration**

  09:30 – 10:30 **Anna Zdunik**
  Thermodynamic formalism for transcendental maps I

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Kurt Falk**
  An introduction to dynamics on hyperbolic manifolds

  12:00 – 14:00 *lunch break*

  14:00 – 14:45 **Mike Todd**
  Continuity of measures

  14:45– 15:15 *break*

  The next presentation is aimed at a general mathematics public.
  15:15 – 16:15 **Krzysztof Barański**
  Ergodic aspects of transcendental dynamics

- **Tuesday, September 29, 2015**

  09:00 – 10:00 **Bogusia Karpińska**
  Thermodynamic formalism for transcendental maps II

  10:00 – 10:30 **Vasiliki Evdoridou**
  Fatous web and non-escaping endpoints

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Peigné / Sambusetti**
  Counting for negatively curved manifolds : an introduction

  12:00 – 14:00 *lunch break*

14:00 – 14:45 **Núria Fagella**
Escaping points in the boundary of Baker domains

14:45 – 15:15 *break*

15:15– 16:00 **Carsten Petersen**
On quasi-conformal (in-) compatibility of satellite copies of the Mandelbrot set

- **Wednesday, September 30, 2015**

09:00 – 10:00 **Trevor Clark**
Rigidity for one-dimensional maps

10:00 – 10:30 **David Marti-Pete**
Escaping Fatou components of transcendental self-maps of the punctured plane

10:30 – 11:00 *coffee / tea break*

11:00 – 12:00 **Genadi Levin**
The Lyapunov exponent of holomorphic maps

12:00 – 14:00 *lunch break*

14:00 – 14:45 **Andrea Sambusetti**
Volume growth and rigidity of negatively curved manifolds of finite volume

14:45 – 15:30 **Marc Peigné**
TBA

15:30– 16:15 *break*

The following talk is the Mathematics Colloquium at the University of Vienna (Meeting room, 3rd floor, Department of Mathematics, University of Vienna, Oskar-Morgenstern-Platz 1).

16:15 – 17:30 **Sebastian van Strien**
Fictitious games: a class of piecewise smooth dynamical system with very interesting properties

- **Thursday, October 1, 2015**

09:00 – 09:45 **Dmitry Dudko**
Self-similarity of the Mandelbrot set around Siegel parameters of periodic type

10:15 – 10:45 **Alexandre Dezotti**
The eventual hyperbolic dimension of entire functions

10:45 – 11:15 *coffee / tea break*

11:15 – 12:00 **Phil Rippon**
Connectedness properties of the set where the iterates of an entire function are unbounded

12:00 – 14:00 *lunch break*

14:00 – 14:45 **Kurt Falk**
Conformal ending measures

14:45 – 15:30 **Neil Dobbs**
Typical behaviour in the exponential family

15:30– 16:00 *break*

16:00 – 16:45 **Trevor Clark**
Quasi-symmetric rigidity for one-dimensional maps

- **Friday, October 2, 2015**

09:00 – 09:45 **Gwyneth Stallard**
Commuting functions and multiply connected wandering domains

10:00 – 10:45 **Jane Hawkins**
Lebesgue measure dynamics of rational maps

10:45 – 11:15 *coffee / tea break*

11:15 – 12:00 **Xavier Jarque**
Wandering domains for composition of entire functions

12:00 – 14:00 *lunch break*

This afternoon will be dedicated to the Budapest-Vienna Seminar.

14:00 – 14:45 **Feliks Przytycki**
Geometric pressure in real and complex 1D dynamics via trees of pre-images and via spanning sets

15:00 – 15:45 **Fanni Selley**
Mean field coupling of identical expanding circle maps

15:45 – 16:15 *break*

16:15 – 17:00 **Mariusz Urbański**
Random dynamics of transcendental meromorphic functions

All talks (with the exception of Van Strien's on Wednesday) take place at the ESI, Boltzmann Lecture Hall!

Abstracts:

Krzysztof Barański
Title: Ergodic aspects of transcendental dynamics
Abstract: In this survey talk we review some elements of the ergodic theory of the dynamics of transcendental entire and meromorphic maps. After presenting a brief introduction to holomorphic dynamics, we will consider ergodic properties of transcendental maps together with some applications to the study of geometry and dimension of the Julia sets and their invariant subsets.

Trevor Clark
Title: Quasi-symmetric rigidity for one-dimensional maps.
Abstract: Rigidity plays a central role in the study of the dynamics of mappings in dimension one. It is the phenomenon that occurs when fairly weak information about a mapping, for example, its combinatorial or topological data, determines metric information about the mapping. In some circumstances, such weak information determines the mapping uniquely. In joint work with Sebastian van Strien, building on earlier work of Sebastian van Strien, Oleg Kozlovski and Weixiao Shen, we have proved quasi-symmetric for a broad class of smooth mappings. In these talks, I will survey some of the tools used to prove quasi-symmetric rigidity and discuss some of its implications.

Alexandre Dezotti
Title: The eventual hyperbolic dimension of entire functions
Abstract: The eventual hyperbolic dimension is introduced as a way to characterize the weight of the hyperbolic sets near infinity. It can be used as a criterion of impediment to the existence of conformal measures. In particular, results concerning the eventual hyperbolic dimension of some Poincar functions will give interesting examples of specific properties in the measurable dynamics of transcendental functions. This is a joint work with Lasse Rempe-Gillen.

Neil Dobbs
Title: Typical behaviour in the exponential family.
Abstract: We shall review recent (and not-so-recent) results concerning maps in the complex exponential family $z \mapsto \lambda \exp(z)$ with good (though non-uniform) expansion properties. For these maps, the Julia set is the whole sphere, so generic orbits are dense. We consider what happens for typical points with respect to Lebesgue measure. Time-permitting, forthcoming results will also be presented.

Dima Dudko
Title: Self-similarity of the Mandelbrot set around Siegel parameters of periodic type.
Abstract: It is indicated by pictures that the Mandelbrot set is self-similar around Siegel parameters of periodic type. Thus we can expect a renormalization operator explaining this phenomenon. In the talk we will discuss a possible candidates. As a preliminary result we will show that certain centers of hyperbolic components "behave as they should" around Siegel parameters. Joint work with Mikhail Lyubich and Nikita Selinger.

Vasiliki Evdoridou
Title: Fatou's web and non-escaping endpoints
Abstract: Let f be Fatou's function, that is, $f(z) = z + 1 + exp(-z)$. We show that the escaping set of f, which consists of all points that tend to infinity under iteration, has a structure known as a spider?s web. We discuss a consequence of this result concerning the non-escaping endpoints of the Julia set of f. More specifically, we

prove that the set of non-escaping endpoints together with infinity form a totally disconnected set.

Núria Fagella
Title: Escaping points in the boundary of Baker domains.
Abstract: We study the dynamical behaviour of points in the boundaries of simply connected Baker domains of meromorphic maps of finite degree on U. We show that there is a dichotomy in terms of the harmonic measure of boundary points that escape to infinity under iteration, which is zero or infinity depending of the type of Baker domain. Additionally, we present some extensions to the infinite degree case and explain how the results also apply to basins of attraction of attracting or parabolic fixed points. This is joined work with K. Barański, X. Jarque and B. Karpińska.

Kurt Falk
1) Title: An introduction to dynamics on hyperbolic manifolds
Abstract: This will be an introductory talk about Hyperbolic Geometry, Kleinian groups and dynamics on hyperbolic manifolds. I will introduce basic notions and problems, and will then give a brief overview of Patterson-Sullivan theory.

2)Title: Conformal ending measures
Abstract: The dynamics of geometrically finite hyperbolic manifolds, where recurrence and ergodicity play a central role, is well understood by means of Patterson-Sullivan theory. For geometrically infinite manifolds or manifolds given by infinitely generated Kleinian groups, non-recurrent dynamics becomes the thick partöf dynamics, not only in measure but often also in (Hausdorff) dimension. Patterson-Sullivan measures are conformal measures at the critical exponent and they work well for divergence type groups; conformal measures at exponents above the critical one were known to exist by earlier work of Sullivan using methods from Harmonic Analysis, but were explicitely constructed only a few years ago by Anderson, Tukia and myself. Such conformal ending measures naturally work well when the Poincare series of the group converges and are thus suitable for studying non-recurrent dynamics in hyperbolic manifolds. In my talk I will present the construction, properties and some first applications of such conformal ending measures.

Jane Hawkins
Title: Lebesgue measure dynamics of rational maps
Abstract: We give a survey about the role of one and two dimensional Lebesgue measure, denoted in complex dynamics. We begin with the classical examples, but move quickly to discuss some more recently studied families of rational maps that preserve finite or infinite measures equivalent to one and two dimensional Lebesgue measure.

Xavier Jarque
Title: Wandering domains for composition of entire functions.
Abstract: In this talk I will present some well-known results on wandering domains for entire transcendental functions. We will focus on Bishop construction of wandering domains for Eremenko-Lyubich class and an application to composite functions. Precisely, we will show that there exist two transcendental entire functions (in Eremenko-Lyubich class) f and g so that neither f nor g have wandering domains but $f \circ g$ and $g \circ f$ have wandering domains.

Bogusia Karpińska
Title: Thermodynamic formalism for transcendental maps II
Abstract: In this survey talk we will review some methods of the thermodynamic formalism which can be used to estimate the Hausdorff dimension of Julia sets (or its significant subsets) for transcendental maps. In parti-

cular we will present some ideas which allow to extend Bowen's formula for certain classes of meromorphic maps.

Genadi Levin
Title: The Lyapunov exponent of holomorphic maps.
This is an introductory survey talk to a recent joint work with Feliks Przytycki and Weixiao Shen. I plan to describe a basic construction and show how to apply it to study the lower exponent. In the opposite direction, for a point with a positive upper exponent we show that many iterates go to a fixed scale with a bounded criticality. Altogether this implies in particular that for holomorphic maps with a single singular value the lower Lyapunov exponent at the singular value is always non-negative and for Lebesgue almost every point in the Julia set of a unicritical polynomial the Lyapunov exponent exists and is equal to zero. I plan to discuss also other applications.

David Marti-Pete
Title: Escaping Fatou components of transcendental self-maps of the punctured plane.
Abstract: We study the escaping set of transcendental self-maps of the punctured plane. The orbits of these points accumulate to zero and/or infinity following what we call essential itineraries. It can be shown that for every essential itinerary, there are points in the Julia set that escape following that itinerary. Therefore, it is a natural question to ask whether there are examples of Fatou components that escape in each possible way as well. Using approximation theory we are able to construct functions with wandering domains and Baker domains that do this. We also study some concrete examples.

Marc Peigné & Andrea Sambusetti
Title: Counting for negatively curved manifolds : an introduction
Abstract: We will consider a negatively curved manifold X with fundamental group G, and will be interested in the asymptotic behavior of the counting function $N(x, y, R)$ of the group G acting on the universal covering of X; for any points $x, y \in X$, the quantity $N(x, y, R)$ denotes here the number of orbits points Gy falling in a ball of center x and radius R. After reviewing some basic notions on the geometry at infinity of such manifolds, we will introduce some classical tools, related to ergodic theory: the Poincaré exponent of G, the volume entropy of X, the Patterson-Sullivan and Bowen-Margulis measures, etc We will then explain some relations between theses invariants and survey some classical results on the dynamic of the geodesic flow on the tangent bundle of X and on counting questions. We will mainly focus on the class of geometrically finite manifolds, which contains in particular all compact or finite volume negatively curved manifolds.

Carsten Petersen
Title : On quasi-conformal (in-) compatibility of satellite copies of the Mandelbrot set.
Abstract: In the groundbreaking paper "On the dynamics of polynomial-like mappings" Douady and Hubbard introduced the notion of polynomial-like maps and they proved the so-called straightening theorem for polynomial-like mappings. To illustrate the power of their theory they used it to identify via straigthening, two types of homeomorphic copies M', primitive and satellite of the Mandelbrot set M inside the Mandelbrot set.

They conjectured that the primitive copies, which are characterized by having a cusp and a root for which the parabolic multiplier is equal to 1, are quasi-conformally homeomorphic to M. This is now a Theorem due to Lyubich. The satellite copies M' however, which are characterized by having a smooth round main component and a root for which the parabolic multiplier is a q-th root of unity for some $q > 1$, Douady and Hubbard conjectured and Lyubich proved to be locally q-c homeomorphic to M away from the root. The natural question is then, if this is an artefact of the proof or if the q.c.-dilatation of the Douady-Hubbard homeomorphism is unbounded near the root. This gives rise to two questions: is the induced Douady-Hubbard homeomorphism onto half of the logistic Mandelbrot set q.c.? Are the induced homeomorphisms between different satellite

copies q.c.?

In this talk I will present a proof that two satellite copies M' and M'' are not q-c homeomorphic, if the root multipliers are q- and q'- roots of unity with $q \neq q$ and in particular no satellite copy is qc-homeomorphic to half of the logistic Mandelbrot set. Joint work with Luna Lomonaco Univ. Of São Paolo.

Feliks Przytycki
Title: Geometric pressure in real and complex 1D dynamics via trees of pre-images and via spanning sets

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Phil Rippon
Title: Connectedness properties of the set where the iterates of an entire function are unbounded
Abstract: This talk will look at connectedness properties of the set of points where the iterates of a transcendental entire function f are unbounded. For example, we show that this set is connected whenever iterates of the minimum modulus of f tend to infinity, and that this property of the minimum modulus holds for many entire functions. We also show that if the set is disconnected, then it has uncountably many components, infinitely many of which are unbounded.

This is joint work with John Osborne and Gwyneth Stallard.

Andrea Sambusetti.
Title: Volume growth and rigidity of negatively curved manifolds of finite volume
Abstract: In the sixties, G. Margulis proved that any Cartan-Hadamard manifold X of negative curvature admitting cocompact lattices has a volume function $v(x,R) = vol(B(x,R))$ which is asymptotic to a purely exponential function: that is, the quotient $v(x,R)/e^{hR}$ tends to a constant m(x) depending on the center x. The exponential growth rate h of $v(x,R)$ is the volume entropy of X, and the function m(x) is called the Margulis' function of X. If $-a^2$ and $-b^2$ are, respectively, upper and lower bounds for the sectional curvature of X, the classical comparison theorems of Riemannian geometry only allow to infer that the volume entropy of X is between $(n-1)a$ and $(n-1)b$, where n is the dimension. The main idea of Margulis was to introduce new powerful tools coming from ergodic theory. Is the limit theorem of Margulis still true for the volume function of spaces of bounded, negative curvature, admitting non-uniform lattices? We prove that for Cartan-Hadamard spaces X with $1/4$-pinched curvature (i.e. $b^2/a^2 \leq 4$) the function $v(x,R)$ is always asymptotic to a pure exponential, and that any non-uniform lattice G of X is of divergence type; this implies in particular that the geodesic flow of any finite-volume, $1/4$-pinched negatively curved manifold always is ergodic and totally conservative with respect to the Bowen-Margulis measure (by Hopf-Tusji-Sullivan theorem). An essential ingredient for this is a generalization to finite-volume manifolds of an entropy-rigidity result of G. Knieper, holding for compact, negatively curved manifolds. On the other hand, we show that for general Cartan-Hadamard spaces admitting non-uniform lattices, with curvature pinched between two negative constants $-a^2$, $-b^2$ with $b^2/a^2 > 4$ (even if arbitrarily close to 4) the volume function $v(x,R)$ can be exponential, lower-exponential or even upper-exponential, depending on the critical exponents of its parabolic subgroups and on the finiteness of the Bowen-Margulis measure. We use a a mixture of tools from classical Riemannian geometry, dynamics and the barycenter method initiated by Besson-Courtois-Gallot.

Fanni Selley
Title: Mean field coupling of identical expanding circle maps

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Gwyneth Stallard
Title: Commuting functions and multiply connected wandering domains.
Abstract: Let f and g be meromorphic functions such that f and g commute. It has been known since Fatou and

Julia that, if f and g are both rational, then their Julia sets must be equal. For transcendental entire functions, however, this is still an open question. Previously, the strongest result was obtained by Bergweiler and Hinkkanen who showed that this is true, provided that neither f nor g has any fast escaping wandering domains. We strengthen this to show that the Julia sets are equal, provided that neither f nor g has any fast escaping simply connected wandering domains. Our proof uses recent results on the properties of multiply connected wandering domains.

This is joint work with Anna Miriam Benini and Phil Rippon.

Sebastian van Strien
Title: Fictitious games: a class of piecewise smooth dynamical system with very interesting properties
Abstract: In economics, biology and game theory, several dynamical systems were introduced to model learning behaviour. One of these systems, best response dynamics (also referred to as fictitious games), was already introduced in the 1950's and models a situation were two players continuously optimise their strategies in order to optimise their payoff. These dynamical systems are piecewise linear, and turn out to have remarkably rich mathematical properties. This talk will aim to give an overview of some recent results, and concentrate on an intriguing relationship between zero-sum games, Hamiltonian dynamics, translation flows in higher dimensions, and a conjecture of Josef Hofbauer.

Mike Todd
Title: Continuity of measures
Abstract: Given a convergent family of interval maps and the associated family of SRB measures, one might hope that the measures would converge to the SRB measure of the limit map. In non-uniformly hyperbolic systems, this naive approach can fail. I will give sharp conditions on precisely when this failure occurs for a very general class of maps. This is part of a wider study of continuity of thermodynamic quantities in collaboration with Neil Dobbs.

Mariusz Urbanski
Title: Random dynamics of transcendental meromorphic functions
Abstract: This is a joint work with M. Volker Mayer It concerns random dynamics of transcendental functions $f : \mathbb{C} \to \overline{\mathbb{C}}$.

We will establish the existence of random conformal measures and their invariant versions. An appropriately defined spectral gap property will be shown. In classical situations there is a natural and powerful proof of this property Which stems from Birkhoff's Contraction Principle for operators preserving a positive cone. This method however fails in our non-compact situation. We will nevertheless define appropriate invariant cones of positive functions and will revive an old approach of Bowen to overcome this difficulty.

Anna Zdunik
Title: Thermodynamic formalism for transcendental maps I
Abstract: In this survey talk we will review some methods of the thermodynamic formalism which can be used to estimate the Hausdorff dimension of Julia sets (or its significant subsets) for transcendental maps. In particular we will present some ideas which allow to extend Bowen's formula for certain classes of meromorphic maps.