DVR 0065528

Programme on

"Modern Theory of Wave Equations"

July 6 - September 30, 2015

organized by

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Workshop I on

"Semi-classical Analysis: Spectral Theory and Resonances"

August 24 - 28, 2015

- **Monday, August 24, 2015**

  09:00 – 09:30 **Opening & Registration**

  09:30 – 10:30 **Steve Zelditch**
  Counting boundary zeros and nodal domains of eigenfunctions

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Semyon Dyatlov**
  Spectral gaps via additive combinatorics

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Jesse Gell-Redman**
  The Feynman Propagator on perturbations of Minkowski space

  15:00 – 15:30 *break*

  15:30 – 16:30 **Hans Christianson**
  Uniform lower bounds for restrictions of quantum ergodic eigenfunctions

- **Tuesday, August 25, 2015**

  09:30 – 10:30 **Fabricio Macia**
  Delocalization of solutions to the Schrödinger equation

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Shu Nakamura**
  High energy asymptotics of the scattering matrix for Schrödinger and Dirac operators

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Stephane Nonnenmacher**
  Logarithmic quasimodes along a hyperbolic orbit

  15:00 – 15:30 *break*

  15:30 – 16:30 **Nicolas Burq**
  Second microlocalization and stabilization of damped wave equations on tori

- **Wednesday, August 26, 2015**

  09:30 – 10:30 **Antonio Sa Barreto**
  Semiclassical Resolvent Estimates on Conformally Compact Manifolds with Variable Curvature at Infinity

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Maciej Zworski**
  Heat traces and inverse problems for scattering resonances

  12:00 – 14:30 *lunch break*

  14:30 – 15:30 **John Toth**
  Nodal Lengths of Steklov-Eigenfunctions on real-analytic Riemannian Surfaces

  15:30 – 16:00 *break*

  16:00 – 17:00 **Dmitri Vassiliev**
  Analysis of first order systems of PDEs on manifolds without boundary

  from 19:00 **HEURIGER**

- **Thursday, August 27, 2015**

  10:00 – 11:00 **Xuwen Zhu**
  Nodal degeneration of hyperbolic metrics and asymptotics of the Weil-Petersson metric on the moduli space

  11:00 – 11:30 *coffee / tea break*

  11:30 – 12:30 **Dean Baskin**
  Asymptotics of scalar waves on asymptotically Minkowski spaces

  12:15 – 14:15 *lunch break*

  14:15 -18:00 **Free Afternoon**

- **Friday, August 28, 2015**

  09:30 – 10:30 **Hiroshi Isozaki**
  Inverse scattering on non-compact manifolds with general metric

  10:30 – 11:00 *coffee / tea break*

  11:00 – 12:00 **Raphael Falcao da Hora**
  Resolvent Estimates on Asymptotically Hyperbolic Manifolds

  12:00 – 14:00 *lunch break*

  14:00 – 15:00 **Martin Vogel**
  Eigenvalue statistics for a class of non-self-adjoint semiclassical differential operators under small random perturbations

  15:00 – 15:30 *break*

  15:30 – 16:30 **Jared Wunsch**
  Semiclassical asymptotics for exterior Helmholtz problems

All talks take place at the ESI, Boltzmann Lecture Hall!

Abstracts:

Dean Baskin: Asymptotics of scalar waves on asymptotically Minkowski spaces

Abstract: In this talk I will describe an asymptotic expansion for solutions of the wave equation on long-range asymptotically Minkowski spacetimes. The exponents seen in the expansion are related to the resonances of an asymptotically hyperbolic problem at timeline infinity. If time permits, I will describe the proof, which also simplifies the short-range setting. This is joint work with András Vasy and Jared Wunsch.

Nicolas Burq: Second microlocalization and stabilization of damped wave equations on tori

Abstract: We consider the question of stabilization for the damped wave equation on tori

$$(\partial_t^2 - \Delta)u + a(x)\partial_t u = 0.$$

When the damping coefficient $a(x)$ is continuous the question is quite well understood and the geometric control condition is necessary and sufficient for uniform (hence exponential) decay to hold. When $a(x)$ is only L^∞ there are still gaps in the understanding. Using second microlocalization (on isotropic submanifolds) we completely solve the question for Damping coefficients of the form

$$a(x) = \sum_{i=1}^{J} a_j 1_{x \in R_j},$$

Where R_j are cubes.

This is joint work with P. Gerard.

Semyon Dyatlov: Spectral gaps via additive combinatorics

Abstract: A spectral gap on a noncompact Riemannian manifold is an asymptotic strip free of resonances (poles of the meromorphic continuation of the resolvent of the Laplacian). The existence of such gap implies exponential decay of linear waves, modulo a finite dimensional space; in a related case of Pollicott–Ruelle resonances, a spectral gap gives an exponential remainder in the prime geodesic theorem.

We study spectral gaps in the classical setting of convex co-compact hyperbolic surfaces, where the trapped trajectories form a fractal set of dimension $2\delta + 1$. We obtain a spectral gap when $\delta = 1/2$ (as well as for some more general cases). Using a fractal uncertainty principle, we express the size of this gap via an improved bound on the additive energy of the limit set. This improved bound relies on the fractal structure of the limit set, more precisely on its Ahlfors-David regularity, and makes it possible to calculate the size of the gap for a given surface.

Hiroshi Isozaki: Inverse scattering on non-compact manifolds with general metric

Abstract: We consider the scattering theory and the related inverse problem on a non-compact Riemannian manifold (or orbifold) $\mathcal{M}$ having the following structure :

$$\mathcal{M} = \mathcal{K} \cup \mathcal{M}_1 \cup \cdots \mathcal{M}_{N+N'},$$

where $\overline{\mathcal{K}}$ is compact, $\mathcal{M}_i$ is diffeomorphic to $(1, \infty) \times M_i$, M_i being a compact $n-1$ dimensional manifold (or orbifold) endowed with the metric h_{M_i}. On each end $\mathcal{M}_i$, the metric of $\mathcal{M}$ is assumed to behave like

$$ds^2 \sim (dr)^2 + \rho_i(r)^2 h_{M_i}, \quad r \to \infty,$$

and $\rho_i(r)$ has the form either $A \exp\big(c_0 r + \beta r^\alpha/\alpha\big),\ 0 < \alpha < 1$, or Ar^β. Moreover, for $1 \le i \le N$, we assume that $c_0 \ge 0$ and if $c_0 = 0$, then $\beta > 0$, and for $N+1 \le i \le N+N'$, we assume $c_0 \le 0$ and if $c_0 = 0$, then $\beta < 0$. This means that the ends $\mathcal{M}_i$ have regular infinities (i.e. with infinite volume) for $1 \le i \le N$, while they have cusps for $N+1 \le i \le N+N'$. We shall solve the Helmholtz equation $(-\Delta_g - \lambda)u = 0$, Δ_g being the Laplace operator on $\mathcal{M}$, and construct a family of generalized eigenfunctions of $-\Delta_g$ which makes it possible to introduce a Fourier transformation on $\mathcal{M}$. By observing the asymptotic behavior of generalized eigenfunctions at infinity, we introduce the S-matrix, and then solve the inverse scattering problem, i.e. the recovery of the manifold $\mathcal{M}$ from one component of the S-matrix (for all energies). One can also consider the inverse scattering from cusp by introducing the generalized S-matrix. This is a joint work with Y. Kurylev and M. Lassas.

Stephane Nonnenmacher: Logarithmic quasimodes along a hyperbolic orbit

Abstract: In the semiclassical limit, it is possible to construct quasimodes of a pseudodifferential operator of order $O(h^\infty)$, which microlocalized along an elliptic periodic orbit, e.g. using Ralston's "Gaussian beams". When the orbit is hyperbolic, the Gaussian beam construction breaks down, due to the exponential spreading along the unstable manifold.

Generalising a recent work of Brooks on hyperbolic surfaces, we consider a 2-dimensional Hamiltonian system exhibiting a hyperbolic periodic orbit γ, and construct quasimodes of order $C\,h/|\log h|$ (thus called logarithmic") which are at least partially localized along γ, in the sense that their associated semiclassical measure contains a singular component $w\delta_\gamma$, where δ_γ is the invariant probability measure on γ. We show that the localization properties depend on the value of the constant $C > 0$: if C is large enough, our quasimode is fully localized along γ, meaning that $w = 1$; on the other hand, for small values of C we can construct quasimodes with a positive weight $0 < w < 1$, which we try to optimize.

Our construction uses a Quantum Normal Form valid near the hyperbolic orbit (derived by Sjöstrand), and proceeds by a time averaging of some initial Gaussian state, up to the Ehrenfest time associated with this orbit.

This is a joint work with Suresh Eswarathasan.

John Toth: Nodal Length of Steklov Eigenfunctions on Real-Analytic Riemannian Surfaces

Abstract: We prove sharp upper and lower bounds for the nodal length of Steklov eigenfunctions on real-analytic Riemannian surfaces with boundary. The argument involves frequency function methods for harmonic functions in the interior of the surface as well as the construction of exponentially accurate approximations for the Steklov eigenfunctions near the boundary (joint with David Sher and Iosif Polterovich).

Dmitri Vassiliev: Analysis of first order systems of PDEs on manifolds without boundary

Abstract: In layman's terms a typical problem in this subject area is formulated as follows. Suppose that our universe has finite size but does not have a boundary. An example of such a situation would be a universe in the shape of a 3-dimensional sphere embedded in 4-dimensional Euclidean space. And imagine now that there is only one particle living in this universe, say, a massless neutrino. Then one can address a number of mathematical questions. How does the neutrino field (solution of the dynamic massless Dirac equation) propagate as a function of time? What are the eigenvalues (stationary energy levels) of the particle? Are there nontrivial (i.e. without obvious symmetries) special cases when the eigenvalues can be evaluated explicitly? What is the difference between the neutrino (positive energy) and the antineutrino (negative energy)? What is the nature of spin? Why do neutrinos propagate with the speed of light? Why are neutrinos and photons (solutions of the Maxwell system) so different and, yet, so similar?

The speaker will approach the study of first order systems of PDEs from the perspective of a spectral theorist

using techniques of microlocal analysis and without involving geometry or physics. However, a fascinating feature of the subject is that this purely analytic approach inevitably leads to differential geometric constructions with a strong theoretical physics flavour.

Steve Zelditch: Counting boundary zeros and nodal domains of eigenfunctions

Abstract: When the billiard dynamics of a surface with boundary are ergodic, the Neumann eigenfunctions have a growing number $n(\lambda)$ of sign-changing zeros on the boundary. By a topological argument, the boundary zeros give rise rise to roughly $1/2n(\lambda)$ nodal domains.

In joint work with J. Jung, we showed that $n(\lambda_j)$ tends to infinity. My talk is mainly about the various identities and estimates that go into the estimate of the number of zeros and into related work: (a) Kuznecov formulae, (b) Completeness formulae; (c) QER theorems; (d)sup norm estimates for Cauchy data, (e) quantum mixing theorems. Joint work with: J. Jung, Christianson-Toth, Han-Hassell-Hezari, Sogge.

Xuwen Zhu: Nodal degeneration of hyperbolic metrics and asymptotics of the Weil-Petersson metric on the moduli space

Abstract: This is joint work with Richard Melrose. We analyze the behavior of the Laplacian on the fibres of a Lefschetz fibration and use it to describe the behavior of the constant curvature metric on a Riemann surface of genus ¿1 undergoing nodal degeneration. In particular this applies to the universal curve over moduli space. The description of the regularity of the fibre hyperbolic metrics, up to the divisors forming the 'boundary' of the Knudsen-Deligne-Mumford compactification of moduli space $\mathcal{M}_{g,n}$, implies boundary regularity for the Weil-Petersson metric.

Maciej Zworski: Heat traces and inverse problems for scattering resonances

Abstract: Suppose that V is a bounded compactly supported potential in odd dimensions. Then solutions of $(\partial_t^2 - \partial_x^2 + V(x))u = 0$ with compactly supported initial data have expansions valid for x in compact sets :

$$u(t,x) = \sum_{j=1}^{N} e^{-i\lambda_j t} u_j(x) + \mathcal{O}(e^{-At}), \;\; \operatorname{Im} \lambda_j \geq -A$$

These λ_j's are called scattering resonances and replace eigenvalues of problems on open domains. It has been known for a while that for a real valued smooth V there are infinitely many resonances (Sá Barreto–Z) and that generic bounded (or smooth) V have resonances saturating the upper bounds on their counting functions (Christiansen–Hislop).

In joint work with Hart Smith we show that any bounded compactly supported potential has to have some resonances. That is done by proving the following (seemingly unknown) result : for V uniformly bounded,

$$t^{\frac{n}{2}} \operatorname{tr}(e^{t(\Delta - V)} - e^{t\Delta}) \in C^\infty([0,\infty)) \iff V \in C^\infty.$$

In fact, we show that potentials with the same set of resonances have to have the same Sobolev regularity which is a modest inverse result.