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FAKULTÄT FÜR MATHEMATIK
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Einladung zur öffentlichen Defensio von

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Thema der Dissertation:

On the Inverse Scattering Transform for the Korteweg-de Vries Equation with Steplike Initial Profile

Abstract: The Korteweg–de Vries (KdV) equation is an integrable wave equation modeling shallow water waves and one of the most prominent soliton equations. The corresponding Cauchy problem was solved by Gardner, Green, Kurskal, and Miura by the inverse scattering transform. In the classical case the initial data will vanish asymptotically and this case is well-understood. Another case, modeling shock and rarefaction waves, is when the initial conditions asymptotically tends to different constants, known as steplike initial conditions.

In the first part of this thesis we study the underlying direct and inverse scattering problem for the one-dimensional Schrödinger equation with steplike potentials. We give necessary and sufficient conditions for the scattering data to correspond to a potential with prescribed smoothness and prescribed spatial decay. This problem has been considered before but our results generalize all previous known results.

In the second part these results are then applied to the Cauchy problem of the KdV equation with steplike initial data. More specifically, we look at the case corresponding to rarefaction waves. For this case we formulate the inverse scattering problem as an oscillatory Riemann–Hilbert factorization problem and apply the nonlinear descent method to determine the long-time behavior of solutions. To analyze the problem one needs to change to a new phase function, the so-called g function, which will depend on a slow variable $\xi = \frac{x}{t}$. After this change the problem can be deformed to an explicitly solvable model problem. Depending on the value of ξ there are three main regions as $t \to \infty$: For $\xi < -\xi_0$ the solution is close to the left constant. For $-\xi_0 < \xi < 0$ there is a rarefaction region where the solution behaves like $\frac{x}{t}$. For $0 < \xi$ there is a soliton region where the solution is given by a sum of solitons.

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