DVR 0065528

Workshop on "Geometry of Computation in Groups"

organized by
Goulnara Arzhantseva (U Vienna),
Olga Kharlampovich (Hunter College, New York), and
Alexei Miasnikov (City College of New York)

March 31 - April 4, 2014

- **Monday, March 31, 2014**

  09:00 **Welcome & Registration**

  10:00 – 11:00 **Gilbert Levitt**
  Vertex and extension finiteness in relatively hyperbolic groups

  11:00 – 11:20 *coffee/tea break*

  11:20 – 12:20 **Jeremy Macdonald**
  Effective coherence, discrimination, and quasi-convexity

  12:30 – 13:30 **Christopher Cashen**
  Growth tight actions

  13:30 – 15:00 *lunch break*

  15:00 – 16:00 **Inna Bumagina**
  The conjugacy and the conjugacy search problem in relatively hyperbolic groups

  16:10 – 17:10 **Alexander Olshanskii**
  Growth and cogrowth in free groups

  17:10 – 17:30 *break*

  17:30 – 18:30 **Zoran Sunic**
  Free groups: from free actions to orders to quasi-characters

- **Tuesday, April 1, 2014**

  09:00 – 10:00 **Martin Bridson**
  Minicourse I, Geometry of residually finite groups: Profinite isomorphism problems

  10:00 – 10:20 *coffee/tea break*

  10:20 – 11:20 **Yves Cornulier**
  Minicourse II: Introduction to the space of marked groups

  11:30 – 12:30 **Markus Lohrey**
  Parallel complexity of the compressed word problem in groups

  12:30 – 14:30 *lunch break*

14:30 – 15:30 **Paul Schupp**
Asymptotic properties of computability

15:40 – 16:40 **Bob Gilman**
Groups and complexity theory

16:40 – 17:00 *break*

17:00 – 18:00 **Damian Osajda**
Graphical small cancellation groups with the Haagerup property

19:30 **Conference Dinner**
at Hotel Palais Strudlhof, Pasteurgasse 1, 1090 Wien. Just 1 minute walk from ESI. Three-course Buffet-style Dinner including one soft drink at EUR 18,– per person to be paid individually [not covered by the ESI].

- **Wednesday, April 2, 2014**

  09:00 – 10:00 **Martin Bridson**
  Minicourse I, Geometry of residually finite groups: Profinite isomorphism problems

  10:00 – 10:20 *coffee/tea break*

  10:20 – 11:20 **Yves Cornulier**
  Minicourse II: Introduction to the space of marked groups

  11:30 – 12:30 **Ashot Minasyan**
  New examples of groups acting on real trees

- **Thursday, April 3, 2014**

  09:00 – 10:00 **Alexei Miasnikov**
  Minicourse I, Geometry of residually finite groups: Exotic residually finite groups

  10:00 – 10:20 *coffee/tea break*

  10:20 – 11:20 **Vincent Guirardel**
  Minicourse II: Introduction to the space of marked groups

  11:30 – 12:30 **Nikolay Romanovskiy**
  Logical aspects of the theory of rigid solvable groups

  12:30 – 14:30 *lunch break*

  14:30 – 15:30 **John S. Wilson**
  Metric ultraproducts of finite simple groups

  15:40 – 16:40 **Swiatoslaw Gal**
  Groups generated by finite set of transformations and biinvariant metrics

  16:40 – 17:00 *break*

  17:00 – 18:00 **Jakub Gismatulin**
  Approximation of groups by manageable structures - weak sofic and weak hyperlinear groups

- **Friday, April 4, 2014**

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  Minicourse I, Geometry of residually finite groups: Dehn Monsters and even worse

  10:00 – 10:20 *coffee/tea break*

  10:20 – 11:20 **Vincent Guirardel**
  Minicourse II: Introduction to the space of marked groups

11:30 – 12:30 **Olga Kharlampovich**
Some algorithms for Γ-limit groups for a torsion-free hyperbolic group Γ

12:30 – 14:30 *lunch break*

14:30 – 15:30 **Alina Vdovina**
Trivalent expanders and Riemann surfaces

15:40 – 16:40 **Laszlo Pyber**
Applications of the product theorem

16:40 – 17:00 *break*

17:00 – 18:00 **François Dahmani**
On the conjugacy problem for some automorphisms of free groups

All lectures take place in the ESI Boltzmann Lecture Hall except for Friday afternoon when they take place in the Schrödinger Lecture Hall

See next pages for the abstracts

ABSTRACTS: Workshop on "Geometry of Computation in Groups"

organized by
Goulnara Arzhantseva (U Vienna),
Olga Kharlampovich (Hunter College, New York), and
Alexei Miasnikov (City College of New York)

March 31 - April 4, 2014

- **Monday, March 31, 2014**

  10:00 – 11:00 **Gilbert Levitt**
  Vertex and extension finiteness in relatively hyperbolic groups

  Given a finitely generated group G, we consider all splittings of G over subgroups in a fixed family, and we discuss whether only finitely many vertex groups appear, up to isomorphism. We also consider isomorphism classes of groups containing G with finite index. — Joint work with Vincent Guirardel.

  11:20 – 12:20 **Jeremy Macdonald**
  Effective coherence, discrimination, and quasi-convexity

  Subgroups are naturally specified by generating sets, but algorithms often require a group presentation. When can we compute the presentation? We prove that this is possible for all groups G that are discriminated by H which is hyperbolic, locally quasi-convex, and torsion-free. Applications include enumeration and algorithmic recognition of such groups G, and effective embedding of G into iterated centralizer extensions of H. — Joint work with I. Bumagin.

  12:30 – 13:30 **Christopher Cashen**
  Growth tight actions

  An action of a group G on a metric space induces an invariant pseudo-metric on the group and on its quotients. The action is growth tight if the exponential growth rate of G with respect to this pseudo-metric is strictly greater than that of every quotient G/N with N infinite. We show that two conditions on the action guarantee growth tightness. The first is that the action contains a strongly contracting element, which means that some group element has an axis that behaves like a geodesic in a hyperbolic space. The second is a technical condition that controls how badly the orbit map distorts the group. The second condition is satisfied if the group action is cobounded, or, more generally, if the action has a quasi-convex orbit. The second condition is also satisfied for the the action of the mapping class group of a hyperbolic surface on its Teichmller space.

  This result generalizes previously known growth tightness results for actions of hyperbolic and relatively hyperbolic groups. We also produce many new examples of growth tight actions. These include CAT(0) groups with rank 1 isometries, mapping class groups acting on their Teichmller spaces, and a family of Brady-Bridson snowflake groups. The latter two families are neither CAT(0) nor relatively hyperbolic. — Joint work with Goulnara Arzhantseva and Jing Tao.

  15:00 – 16:00 **Inna Bumagina**
  The conjugacy and the conjugacy search problem in relatively hyperbolic groups

  If u and v are two conjugate elements in a hyperbolic group then the bound on the length of a shortest conjugating element is linear in terms of the lengths of u and v; this was shown by Lysenok in 1989. This estimate leads to an obvious algorithm to solve both the conjugacy and the conjugacy search problems in hyperbolic groups; however, the algorithm has exponential time complexity.

  In the book by Bridson and Haefliger one finds a polynomial time algorithm to solve the conjugacy problem. Their proof also provides a linear bound on the length of a shortest conjugating element. An elaborate version of the algorithm, due to Epstein and Holt, allows one to solve the conjugacy problem in linear time. I will explain how these results generalize to relatively hyperbolic groups.

16:10 – 17:10 **Alexander Olshanskii**
Growth and cogrowth in free groups

We study the asymptotic behavior of growth and cogrowth functions of the subgroups in free groups. Some relevant results shall also be presented in my talk. In particular, I will formulate theorems on the actions of maximal growth and on pairs of finitely generated subgroups in free groups.

17:30 – 18:30 **Zoran Sunic**
Free groups: from free actions to orders to quasi-characters

We utilize a criterion for the existence of a free subgroup acting freely on at least one of its orbits to construct such actions of the free group on the circle and on the line, leading to orders on free groups that are particularly easy to state and work with. The dynamical realization of the obtained orders is very flexible and allows for modifications, which yield Cantor sets of orders on the free group. We then switch to a restatement of the orders in terms of certain quasi-characters of free groups, from which properties of the defined orders may be deduced (some have positive cones that are context-free, some have word reversible cones, some of the orders extend the usual lexicographic order, and so on).

- **Tuesday, April 1, 2014**

09:00 – 10:00 **Martin Bridson**
Minicourse I, Geometry of residually finite groups: Profinite isomorphism problems

In this mini course I shall focus primarily on recent joint work with Henry Wilton. I shall give a thorough outline of the proof that there is no algorithm that, given a finitely presented group, can determine whether or not the group presented has a non-trivial finite quotient, and I shall explain why this lack of decidability persists in the setting of compact non-positively curved 2-complexes. I shall then explain how a topological improvement of this theorem can be used to prove that there is no algorithm that, given a pair of finitely presented residually finite groups $P \leq G$ can determine whether or not the inclusion map induces an isomorphism of profinite completions, and I shall explain the resolution of a conjecture of Peter Cameron concerning the developability of finite sets of partial permutations.

References:
`arXiv:1401.2790` The isomorphism problem for profinite completions of residually finite groups (Bridson, Wilton)
`arXiv:1401.2273` The triviality problem for profinite completions (Bridson, Wilton)
`arXiv:1401.2273` The triviality problem for profinite completions (Bridson)

10:20 – 11:20 **Yves Cornulier**
Minicourse II: Introduction to the space of marked groups

In the first hour, I will introduce to the space of marked groups. This is a compact topology on the set of groups endowed with a finite generating family. An important feature of this topology is that the neighborhood of a given marked group only depends on the isomorphism class of the underlying group.

In the second lecture, I will address various fixed point properties: it is natural to ask whether a the negation of such a property is a closed property in the space of groups. For instance, Gromov proved that if (X) is a class of metric spaces closed under scaling ultralimits, then the class of groups having a fixed-point-free action on some metric space in (X) is closed in the space of marked groups.

11:30 – 12:30 **Markus Lohrey**
Parallel complexity of the compressed word problem in groups

The compressed word problem for a finitely generated group G asks, whether a given compressed word over the generators of the group evaluates to the group identity. For the compression of words, straight-line programs, i.e., context-free grammars that produce a single word, are used. Such a straight-line program can be also viewed as a circuit (a directed acyclic graph), where the leaves are labelled with group generators and internal nodes compute the product of their children in left-to-right order. It is known that the (ordinary) word problem for a group G can be reduced in polynomial time to the compressed word problem for G.

For many important classes of groups the compressed word problem can be solved in polynomial time. Important examples are virtually special groups and hyperbolic groups (for the latter class, the result is due to Saul Schleimer). In recent years also complexity classes below P (polynomial time) have received attention in computational group theory. In this context, the class NC turns out to be important. It is the class of all problems that can be solved in polylogarithmic time with polynomially many processors, and intuitively it corresponds to the class of all problems that can be efficiently solved on a parallel computer. This leads to the question, for which groups the compressed word problem belongs to NC. For finite groups the answer was given by Beaudry, McKenzie, Peladeau, and Therien: If a finite group G is solvable, then the compressed word problem belongs to NC, otherwise it is complete for the class P. In the talk, I will show that for finitely generated nilpotent groups, the compressed word problem belongs to NC. Actually, this results extends to groups that are (finitely generated nilpotent)-by-(finite solvable). Currently, this is the only known class of groups with a compressed word problem in NC.

14:30 – 15:30 **Paul Schupp**
Asymptotic properties of computability

In this talk I will try to explain how the asymptotic-generic point of view of geometric group theory has led to the development of a new area of computability theory. I will discuss some main ideas of äpproximate computability generic computability, coarse computability and computability at densities less than 1.

15:40 – 16:40 **Bob Gilman**
Groups and complexity theory

Well known results on the unsolvability of decision problems for finitely presented groups prefigured a strong connection between combinatorial group theory and complexity theory. We will talk about some recent developments in this area.

17:00 – 18:00 **Damian Osajda**
Graphical small cancellation groups with the Haagerup property

We prove the Haagerup property (= Gromov's a-T-menability) for finitely generated groups defined by infinite presentations satisfying the graphical $C'(\overline{\lambda})$–small cancellation condition with respect to graphs endowed with a compatible wall structure. We deduce that these groups are coarsely embeddable into a Hilbert space and that the strong Baum-Connes conjecture and, hence, the Baum-Connes conjecture with arbitrary coefficients hold for them. As the main step we show that $C'(\overline{\lambda})$–complexes satisfy the linear separation property. Our result provides many new examples and a general technique to show the Haagerup property for graphical small cancellation groups. — Joint work with Goulnara Arzhantseva.

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- **Wednesday, April 2, 2014**

  09:00 – 10:00 **Martin Bridson**
  Minicourse I, Geometry of residually finite groups: Profinite isomorphism problems

  See above for the abstract and the references of this minicourse.

10:20 – 11:20 **Yves Cornulier**
Minicourse II: Introduction to the space of marked groups

See above for the abstract of this minicourse.

11:30 – 12:30 **Ashot Minasyan**
New examples of groups acting on real trees

A real tree is a geodesic 0-hyperbolic metric space, i.e., a geodesic metric space in which every geodesic triangle is a tripod. In 1987 P. Shalen asked whether any finitely generated group which has a non-trivial (i.e., without a global fixed point) isometric action on a real tree also admits a non-trivial simplicial action on a simplicial tree. Since then several authors (including Rips, Bestvina-Feighn, Sela, Guirardel, and others) showed that the answer is affirmative if one imposes additional assumptions on the behavior of arc stabilizers. During the talk I will discuss a construction which allows to produce counterexamples to Shalen's question in its original form.

- **Thursday, April 3, 2014**

09:00 – 10:00 **Alexei Miasnikov**
Minicourse I, Exotic residually finite groups: Finitely presented residually finite groups, Dehn functions, and depth

In this lecture I will address the following principle questions for finitely presented residually finite groups G: how large could be the Dehn function of G? How large could be the gap between the complexity of the word problem and the Dehn functions of G? What is the time complexity of the classical McKinsey algorithm for the word problem in G (this is the only known uniform algorithm for the word problem in such groups)? How deep could be the depth functions in G? The depth function measures how deep one has to go into finite index subgroups to separate an element of a given length in G from the identity. — Based on joint results with O. Kharlampovich and M. Sapir.

10:20 – 11:20 **Vincent Guirardel**
Minicourse II: Introduction to the space of marked groups

We will look at limits of hyperbolic groups in the space of marked groups from two points of views. First, we will look at the set of groups that are accumulations of hyperbolic groups, and show that most such groups are rather wild. Second, we will fix a hyperbolic group (the free group will be our preferred example), and look at all groups obtained at the limit by varying the generating systems of this fixed group. The class of thus obtained limit groups is quite tame, in contrast to the first situation.

11:30 – 12:30 **Nikolay Romanovskiy**
Logical aspects of the theory of rigid solvable groups

A group G is said to be m*-rigid* if it has a normal series

$$G = G_1 > G_2 > \ldots > G_m > G_{m+1} = 1$$

with abelian factors each of which G_i/G_{i+1}, viewed as an $\mathbb{Z}[G/G_i]$-module, has no torsion. For example free solvable groups are rigid. A rigid group G is called *divisible* if any factor G_i/G_{i+1} is a divisible module over the ring $\mathbb{Z}[G/G_i]$ or, in other words, it is a vector space over skew field of fractions of this ring.
We say for m-rigid groups that H is embedded into G *independently* if any system elements of H_i/H_{i+1} linear independent over the ring $\mathbb{Z}[H/H_i]$ has to be linear independent over the ring $\mathbb{Z}[G/G_i]$.

Theorem 1 *Arbitrary m-rigid group can be embedded independently into some divisible m-rigid group.*

Malcev proved that a free solvable group of length ≥ 2 has undecidable elementary theory. The universal theory of a free metabelian group is decidable (Chapuis).

Theorem 2. *The universal theory of a free solvable group of length ≥ 4 is undecidable.*

For the class Σ_m of rigid groups of length $\leq m$ we define algebraic closed objects: G is called *algebraic* *closed* if for any independent embedding $G \hookrightarrow H$ in this class any system of equations over $x_1, \dots, x_n$ with coefficients from G has a solution in G^n if and only if it has a solution in H^n. G is called *existential closed* if for any such embedding any $\exists$-formula is true on G if and only if it is true on H.

Theorem 3. *Divisible m-rigid groups = algebraic closed objects in Σ_m = existential closed objects in Σ_m.*

We study elementary theories of divisible m-rigid groups and construct a system of axioms in group theory signature which defines exactly all divisible m-rigid groups. Denote by $\mathfrak{T}_m$ corresponding theory. Fix some countable divisible m-rigid group M. We prove that this group is constructible. Extend the signature of group theory by constants from M. We add to $\mathfrak{T}_m$ some recursive system of axioms which means that M is embedded independently into given rigid group. Denote corresponding theory by $\mathfrak{T}_m(M)$.

Theorem 4. *The theories $\mathfrak{T}_m$ and $\mathfrak{T}_m(M)$ are complete and therefore decidable.*

Theorems 3 and 4 were proved joint with Alexei Myasnikov.

14:30 – 15:30 **John S. Wilson**
Metric ultraproducts of finite simple groups

Metric ultraproducts of structures have arisen in a variety of contexts. The study of the case when the structures are finite groups is recent and motivated partly by the connection with sofic groups. We report on current joint work with Andreas Thom on the topological and algebraic properties of metric ultraproducts of finite simple groups.

15:40 – 16:40 **Swiatoslaw Gal**
Groups generated by finite set of transformations and biinvariant metrics

Abstract

17:00 – 18:00 **Jakub Gismatulin**
Approximation of groups by manageable structures - weak sofic and weak hyperlinear groups

I will report some recent results on groups with good metric approximation properties, called weak sofic and weak hyperlinear groups. The class of weak sofic groups was introduced by L. Glebsky and L. M. Rivera, as a generalization of the notion of a sofic group, defined by B. Weiss and M. Gromov. Sofic and hyperlinear groups can be characterized as subgroups metric ultraproducts of families of certain metric groups: symmetric groups with the Hamming distance and unitary groups of finite rank with the Hilbert-Schmidt distance. In fact, there is another characterization of sofic groups: a group is weak sofic if an only if it can be embedded into an abstract quotient of a profinite group. Therefore one can define and study the class of weak hyperlinear groups: a group is weak hyperlinear if and only if it can be embedded into an abstract quotient of a compact Hausdorff group.

- **Friday, April 4, 2014**

09:00 – 10:00 **Alexei Miasnikov**
Minicourse I, Exotic residually finite groups: Dehn Monsters and even worse

I will discuss finitely generated recursively presented residually finite groups with really strange algorithmic properties. To build them we need Golod-Shafarevich construction and a forcing-type argument from logic. — Based on a joint work with D. Osin and B. Khoussainov.

10:20 – 11:20 **Vincent Guirardel**
Minicourse II: Introduction to the space of marked groups

See above for the abstract and the references of this minicourse.

11:30 – 12:30 **Olga Kharlampovich**
Some algorithms for Γ-limit groups for a torsion-free hyperbolic group Γ

Γ-limit groups may not be finitely presented. Indeed, finitely generated subgroups of Γ may not be finitely presented. We will show how to work algorithmically with Γ-limit groups. In particular we will show how to obtain a finite presentation of maximal limit quotients of a group relative to a finite number of finitely generated subgroups of Γ, how to find the embedding of Γ-limit group into an NTQ group and solve other problems needed for the decidability of the elementary theory of Γ. If time permits, we will also sketch how to prove the decidability of the AE theory of Γ. — These are joint results with A. Myasnikov.

14:30 – 15:30 **Alina Vdovina**
Trivalent expanders and Riemann surfaces

We introduce a family of trivalent expanders which tessellate compact hyperbolic surfaces with large isometry groups. We compare this family with Platonic graphs and modifications of them and prove topological and spectral properties of these families. This is joint work with Ioannis Ivrissimtzis and Norbert Peyerimhoff.

15:40 – 16:40 **Laszlo Pyber**
Applications of the product theorem

The Product Theorem for finite simple groups was obtained independently by Breuillard-Green-Tao and Pyber-Szab around 2010. It says that if A is a generating subset of a group L of Lie type of bounded rank then AAA is much larger than A unless it is L itself. The theorem has applications to estimating the diameters of Cayley graphs and orbital graphs, to the construction of expanders and more. We will consider these applications and possible extensions of the Product Theorem.

17:00 – 18:00 **François Dahmani**
On the conjugacy problem for some automorphisms of free groups

Let F be a free group. By studying isomorphisms between semi direct product with Z, that are hyperbolic, or relatively hyperbolic of a certain kind, we investigate the conjugacy problem for some outer automorphisms of F. A main tool is to understand some orbit problem by considering the relevant JSJ decomposition. Another tool is the body of works on the isomorphism problem in different contexts.