ESI Anniversary - Two Decades at the Interface of Mathematics and Physics

The [Un]reasonable Effectiveness of Mathematics in the Natural Sciences

at the ESI, Vienna, April 29 - 30, 2013

- **Monday, April 29, 2013**

  9:00 a.m. Opening Words
  Heinz Engl, Rector, University of Vienna
  Joachim Schwermer, Director, ESI
  Marta Sanz-Solé, President, EMS

  9:30 a.m.
  David Ruelle (IHES, Bures-sur-Yvette, France)
  Post-human mathematics

  11:00 a.m.
  Sylvia Serfaty (U Paris 6, Paris, France)
  Towards the Abrikosov lattice in the Ginzburg-Landau model of superconductivity

  2:00 p.m.
  Alain Connes (Collège de France, Paris, France)
  Variability, time and the quantum

  3:15 p.m.
  Michael Douglas (State U of New York, Stony Brook, USA)
  String theory and the real world

  5:00 p.m.
  Jeremy Gray (The Open University, Milton Keynes, UK)
  "The soul of the fact" – Poincaré and proof

  7:00 p.m.
  Reception at the Erwin Schrödinger International Institute (ESI)

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- **Tuesday, April 30, 2013**
  9:30 a.m.
  Reinhard Werner (Leibniz U, Hannover, Germany)
  How spectral properties may be irrelevant in the long run

  11:00 a.m.
  Wolfgang Lück (Hausdorff Institute, Bonn, Germany)
  An introduction to L^2-Betti numbers and their applications

  2:00 p.m.
  Yuri Tschinkel (Courant Institute, New York, USA)
  Diophantine equations and their hidden symmetries

  3:30 p.m.
  Peter Goddard (Institute for Advanced Study, Princeton, USA)
  Algebras, groups, and strings

All lectures take place at the Boltzmann Lecture Hall, Erwin Schrödinger International Institute for Mathematical Physics, Boltzmanngasse 9, 1090 Vienna, Austria

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ABSTRACTS

David Ruelle [IHES, Bures-sur-Yvette, France]
Post-human mathematics
Abstract: Present day mathematics is a human construct, where computers are used more and more but do not play a creative role. This situation may change however: computers may become creative, and since they function very differently from the human brain they may produce a very different sort of mathematics. We discuss what this post-human mathematics may look like, and the philosophical consequences that this may entail.

Sylvia Serfaty [U Paris 6, Paris, France]
Towards the Abrikosov lattice in the Ginzburg-Landau model of superconductivity
Abstract: In recent years there has been significant mathematical progress in the understanding of the Ginzburg-Landau model of superconductivity, from its rigorous derivation from quantum many-body theory, to the analysis of phase transitions with appearance of lattices of quantized vortices, the famous Abrikosov lattices. I will review some of these results and mathematical challenges, as well as connections with classical Coulomb systems.

Alain Connes [Collège de France, Paris, France]
Variability, time and the quantum
Abstract: I will explain how the mathematical notion of "real variable" is best encoded by the quantum mechanical formalism of self-adjoint operators in Hilbert space. After this mathematical preliminary I will explain how one can conceive of the flow of time as emerging, using the Kubo-Martin-Schwinger condition, from one fundamental aspect of the quantum world which is intrinsic and irreducible variability.

Michael Douglas [State U of New York, Stony Brook, USA]
String theory and the real world
Abstract: Superstring theory is the leading candidate for a theory unifying the fundamental interactions of nature. Yet, after almost 30 years of work, we have no clear experimental evidence for or against the claim that it describes our world. Why do we believe it? What are the prospects for testing it?

Jeremy Gray [The Open University, Milton Keynes, UK]
"The soul of the fact" – Poincaré and proof
Abstract: Throughout his working life Henri Poincaré was concerned to promote the understanding of mathematics and physics. This is as apparent in his views about geometry, his conventionalism, and his theory of knowledge, as it is in his work on electricity and optics, on number theory, and function theory. This talk will argue that this is one of the ways Poincaré discharged his responsibilities as a scientist, and that it accounts not only for a surprising degree of unity in his work but also gives it its distinctive character – at once profound and elusive.

Reinhard Werner [Leibniz U, Hannover, Germany]
How spectral properties may be irrelevant in the long run
Abstract: A simple model system is presented in which the spectral type depends very sensitively on a parameter: pure point, absolutely continuous and singular spectrum each hold for a dense set of values, although all finite time expectations depend continuously on the parameter. Therefore any spectral type is consistent with the dynamics in any long (but finite) run. Of course, this merely points out the lack of exchangeability of the infinite time limit and limits in the parameter, which is a fairly common occurrence. Related phenomena are the lack of direct relevance of ergodicity for establishing approach to equilibrium and the high (quantum-) computational complexity of finding the ground state of a large quantum system as opposed to the low complexity of simulating the dynamics. With regard to the last problem one may well ask: If Nature does not find the ground state on any reasonable time scale, why should I care about it? This poses the challenge to come up with mathematical notions which have more to say about quantum dynamics on finite time scales.

Wolfgang Lück [Hausdorff Institute, Bonn, Germany]
An introduction to L^2-Betti numbers and their applications
Abstract: Betti numbers of closed manifolds are classical invariants in topology. Atiyah proposed a generalization, called L^2-Betti numbers, for the universal covering of closed manifolds taking the operation of the fundamental group into account. These invariants have analytic interpretations in terms of the heat kernel as well as topological interpretations in terms of simplicial homology. They can also be defined for more general spaces and for groups. They have striking applications to various prominent beautiful problems in differential geometry, topology and group theory, where on the first glance these invariants do not seem to appear. We will discuss a selection of such problems indicating where the L^2-Betti numbers occur as an important tool in their solutions.

Yuri Tschinkel [Courant Institute, New York, USA]
Diophantine equations and their hidden symmetries
Abstract: The structure of solutions of diophantine equations is often governed by symmetries invisible from the shape of the equations. I will discuss representative examples of this phenomenon, involving linear algebraic groups and Galois groups.

Peter Goddard [Institute for Advanced Study, Princeton, USA]
Algebras, groups, and strings
Abstract: Aspects of the interweaving development of the study of infinite-dimensional algebras, such as the Virasoro algebra and Kac-Moody algebras, and of string theory will be reviewed to illustrate the symbiotic relationship between mathematics and theoretical physics.