Parallel computing to study numerically the Davey-Stewartson II equation.

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The Davey-Stewartson systems read

$$\begin{aligned} i\epsilon u_t + \epsilon^2 u_{xx} - \alpha\epsilon^2 u_{yy} + 2\rho\left(\phi + |u|^2\right)u &= 0 \\ \phi_{xx} + \alpha\phi_{yy} + 2\,|u|^2_{xx} &= 0 \end{aligned} \tag{1}$$

where α, $\rho = \pm 1$, $\epsilon \ll 1$ is a small dispersion parameter, and ϕ represents the velocity potential of the fluid. The case $\rho = -1$ corresponds to the focusing case and $\rho = 1$ to the defocusing case.
These systems appear in many physical applications, for example in the study of nonlinear water waves [1] and in plasma physics [4], [5].

Two main questions are addressed in this talk, first the formation of singularities (blow up), and the semiclassical limit of the Davey-Stewartson II equation (that is the case $\alpha = 1$). More precisely, we first study blow-up in solutions to the focusing Davey-Stewartson II equation by studying perturbations of exact solutions as the lump and the Ozawa solution, see [3]. We then look at the semiclassical limit of the Davey-Stewartson II equation, which shows a similar behavior as the nonlinear Schrödinger equation (NLS) in (1+1) dimensions, and which is characterized by the appearance of a zone of rapid modulated oscillations in the solutions and the possibility of blowup. So far there is no theory of there are no analytic prediction about solutions of (1) in the semiclassical limit, and it is unclear whether there will be blow-up in this case. Numerical studies are supposed to provide more insight into these questions.

Due to the modulational instability of the focusing DS II equation, high resolution simulations are needed, and can be achieved by using parallel computing. We developed parallel FORTRAN 90 codes, which compute numerical solutions of DS II system by using successively, a semi-discretization in spatial coordinates in Fourier space, and then time-stepping schemes of order 4, which have been found [2] to be very efficient in this context.

References

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