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ESI SENIOR RESEARCH FELLOW LECTURES

Winter Term 2011

The Erwin Schrödinger International Institute of Mathematical Physics (ESI) of the University of Vienna offers the following lectures held by Senior Research Fellows in residence during the winter term 2011. For more information and related literature please visit the ESI home page www.esi.ac.at

L-functions and Functoriality
Prof. James W. Cogdell (Dept. Mathematics, Ohio State University, Columbus, U.S.A.)
Wednesday, 11 a.m - 1 p.m., 2 p.m. - 3 p.m.
Course begin: November 9, 2011
ESI, Erwin Schrödinger Lecture Hall

Abstract:
The principle of functoriality is one of the central tenets of the Langlands program; it is a purely automorphic avater of Langlands vision of a non-abelian class field theory. There are two main approaches to functoriality. The one envisioned by Langlands is through the Arthur-Selberg trace formula, and with the recent work of Ngo, Arthur, and others this is now becoming available. The second method is that of L-functions as envisioned by Piatetski-Shapiro and is based on the converse theorem for GL(n).In this series of lectures I would like to explain the L-function approach to functoriality and how it has been applied.

I will begin with some basic material on automorphic forms and representations, primarily for GL(n). Then I will spend a number of lectures developing the theory of integral representations for Rankin-Selberg L-functions for GL(n)xGL(m), up to and including the converse theorems for GL(n). The converse theorem in this context gives a way of telling when a representation of GL(n) is automorphic in terms of the analytic applications of its twisted L-functions.

To apply the converse theorem one must control the analytic properties of L-functions. There are two principal ways to do this. One is the method of integral representations, as we will have discussed for GL(n). The other is the Langlands-Shahidi method, which understands L-functions through the Fourier coefficients of Eisenstein series. As we will need this for our applications, I will spend a few lectures surveying this theory.

Finally, I will explain the local and global Langlands conjectures and the formulation of Langlands' principal of functoriality. I will discuss how one can use the converse theorem for GL(n) as a vehicle to obtain functoriality to GL(n) and then implement this for the liftings from classical groups to GL(n) and also the symmetric power liftings for GL(2). These symmetric power liftings and their variants give the best general bounds towards the Ramanjan conjectures for GL(2), and I will end by explaining this.

Joachim Schwermer
Director, ESI