

ESF-EMS-ERCOM Conference on  
Completely Integrable Systems and Applications

Chair: Gerald Teschl  
Cochair: Spyros Kamvissis

ESI Vienna  
3.–8. July 2011

Mon 4.7		Tue 5.7		Wed 6.7		Thu 7.7	
8:45–9:00	Opening						
9:00–9:45	Session 1 Chair: Spyros Kamvissis Athanasios S. Fokas	Session 5 Chair: Athanasios S. Fokas Helge Holden	Session 9 Chair: Fritz Gesztesy Anne Boutet de Monvel	Session 11 Chair: Anne Boutet de Monvel Ira Egorova			
9:45–10:30	Jonatan Lenells	Adrian Constantin	Dmitry Shepelsky	Spyros Kamvissis			
		Coffe break					
11:00–11:45	Session 2 Chair: Pierre Van Moerbeke	Session 6 Chair: Stephanos Venakides Tamara Grava	Session 10 Chair: Jerry Bona Arno Kuijlaars	Session 12 Chair: Gerald Teschl Pierre Van Moerbeke			
11:45–12:30	Beatrice Pelloni Jerry Bona	Stephanos Venakides	Ravi Srinivasan	Fritz Gesztesy			
		Lunch break					
14:30–14:55	Session 3 Chair: Arno Kuijlaars	Session 7 Chair: Stephanos Venakides Aleksi Rybkin	Excursion	Session 13 Chair: Tamara Grava Dong Wang			
14:55–15:20	Jinho Baik	Peter D. Miller		Reinout Quispel			
15:20–15:45	Mattia Cafasso	Alexander Minakov		Jianguang Wang			
15:45–16:10	Hongqiu Chen Yen Do	Antonio Moro		Viktor Enolski			
		Coffe break					
16:40–17:05	Session 4 Chair: Beatrice Pelloni	Session 8 Chair: Jonatan Lenells	Session 14 Chair: Ira Egorova				
17:05–17:30	Aleksey Kostenko	Alexander Soshnikov	Dionyssios Mantzavinos				
17:30–17:50	Irina Nenciu	Alexander Sakhnovich	Davide Masoero				
17:50–18.10	Katrin Grunert	Johanna Michor	Hans Lundmark				
18.10–18.30	Boris Konopelchenko	Jacek Szmigielski	Alice Mikikits-Leitner				
	Iasonas Hitzazis	TBA	Caroline Kalla				
	Dinner	Forward Look	Poster				

## Detailed Program

All events will take place at the ESI (Boltzmanngasse 9, A-1090 Vienna) unless otherwise indicated.

### Sunday, July 3

17.00 onwards	Registration at the ESF desk
19.00	Welcome Drink and Reception

## Monday, July 4

08.45–09.00	Conference Opening	
<b>Session 1</b>	Chair: Spyros Kamvissis, University of Crete	
09.00–09.45	Athanasios S. Fokas, Cambridge	p11
	A unified and effective method for integrable nonlinear PDEs	
09.45–10.30	Jonatan Lenells, Baylor University	p16
	Boundary value problems for the Ernst equation	
10.30–11.00	Coffee break	
<b>Session 2</b>	Chair: Pierre Van Moerbeke, UCL Louvain	
11.00–11.45	Beatrice Pelloni, University of Reading	p23
	Boundary value problems for a nonlinear elliptic PDE: the elliptic sine-Gordon equation	
11.45–12.30	Jerry Bona, University of Illinois at Chicago	p??
	Coupled systems of KdV Equations	
12.30–14.00	Lunch break	
<b>Session 3</b>	Chair: Arno Kuijlaars, Katholieke Universiteit Leuven	
14.30–14.55	Jinho Baik, University of Michigan	p8
	Complete matchings and random matrix theory	
14.55–15.20	Mattia Cafasso, Centre de Recherches Mathématiques	p9
	Two transitions between the Pearcey and the Airy process	
15.20–15.45	Hongqiu Chen, University of Memphis	p9
	Long-wave limit of periodic solutions of nonlinear wave equations	
15.45–16.10	Yen Do, Georgia Institute of Technology	p10
	Oscillatory Riemann–Hilbert problems and stationary phase	
16.10–16.40	Coffee break	
<b>Session 4</b>	Chair: Beatrice Pelloni, University of Reading	
16:40–17.05	Aleksey Kostenko, Institute of Applied Mathematics and Mechanics NASU	p16
	Long-time asymptotics for the Camassa–Holm equation	
17:05–17.30	Irina Nenciu, University of Illinois at Chicago	p23
	The periodic Ablowitz–Ladik equation and Floquet CMV matrices	
17.30–17.50	Katrin Grunert, University of Vienna, AT	p13
	A Lipschitz metric for the Camassa–Holm equation	
17.50–18.10	Boris Konopelchenko, University of Salento	p15
	Harrison cohomology and integrable systems for the hyperelliptic subsets in Birkhoff strata of the Sato Grassmannian	
18.10–18.30	Iasonas Hitzazis, University of Patras	p13
	Initial-boundary problems for integrable nonlinear PDEs	
19.00–20.00	Forward Look Plenary Discussion	

## Tuesday, July 5

<b>Session 5</b>	Chair: Athanassios S. Fokas, Cambridge	
09.00–09.45	Helge Holden, NTNU Trondheim	p14
	On the nonlinear variational wave equation	
09.45–10.30	Adrian Constantin, University of Vienna	p9
	The hydrodynamical relevance of the Camassa–Holm and Degasperis–Procesi equations	
10.30–11.00	Coffee break	
<b>Session 6</b>	Chair: Stephanos Venakides, Duke University	
11.00–11.45	Tamara Grava, SISSA Trieste	p12
	Universality in Hamiltonian PDEs	
11.45–12.30	Stephanos Venakides, Duke University	p??
	Steepest descent for integrable systems	
12.30–14.00	Lunch break	
<b>Session 7</b>	Chair: Helge Holden, NTNU Trondheim	
14.30–14.55	Aleksei Rybkin, University of Alaska Fairbanks	p24
	The KdV equation, the Titchmarsh–Weyl $m$ -function and Hankel operators	
14.55–15.20	Peter D. Miller, University of Michigan	p20
	Fluxon condensates and the semiclassical sine-Gordon equation	
15.20–15.45	Alexander Minakov, Institute for Low Temperature Physics	p21
	Long-time behavior of the solution to the mKdV equation with the step like initial data	
15.45–16.10	Antonio Moro, SISSA	p22
	Thermodynamic phase transitions and shock singularities	
16.10–16.40	Coffee break	
<b>Session 8</b>	Chair: Jonatan Lenells, Baylor University	
16:40–17.05	Alexander Soshnikov, University of California at Davis	p25
	Random matrices: finite rank deformations and fluctuations of matrix entries of regular functions	
17:05–17.30	Alexander Sakhnovich, University of Vienna	p24
	Weyl theory, inverse spectral transform, and initial-boundary value problems for integrable equations	
17.30–17.50	Johanna Michor, University of Vienna	p19
	On the spatial asymptotics of solutions of the Ablowitz–Ladik hierarchy	
17.50–18.10	Jacek Szmigielski, University of Saskatchewan	p26
	On an inverse spectral problem associated to a two-potential generalization of the Degasperis–Procesi equation	
18.10–18.30	TBA, -	p??
	TBA	
20.00	Evening reception by the mayor of Vienna Heuriger S’Pfiff, Rathstraße 4, 1190 Vienna	

## Wednesday, July 6

<b>Session 9</b>	Chair: Fritz Gesztesy, UMC Columbia	
09.00–09.45	Anne Boutet de Monvel, University Paris Diderot Long-time dynamics of step-like data for NLS+	p8
09.45–10.30	Dmitry Shepelsky, Institute for Low Temperature Physics The short-wave model for the Camassa–Holm equation: the Riemann–Hilbert approach	p24
10.30–11.00	Coffee break	
<b>Session 10</b>	Chair: Jerry Bona, University of Illinois at Chicago	
11.00–11.45	Arno Kuijlaars, Katholieke Universiteit Leuven Multiple orthogonal polynomials in the normal matrix model	p16
11.45–12.30	Ravi Srinivasan, University of Texas at Austin Burgers turbulence, kinetic theory of shock clustering, and complete integrability	p25
12.30–14.00	Lunch break	
<b>Excursion</b>	Wachau <a href="http://en.wikipedia.org/wiki/Wachau">http://en.wikipedia.org/wiki/Wachau</a>	
14:00	Bus to Melk	
15:30	Ship from Melk to Krems	
17:00	Bus from Krems to Palt	
17:45	Winzerhof Rosenberger	
21:30	Bus back to Vienna	
22:45	Arrival in Vienna	

## Thursday, July 7

<b>Session 11</b>	Chair: Anne Boutet de Monvel, University Paris Diderot	
09.00–09.45	Ira Egorova, Institute for Low Temperature Physics Inverse scattering transform for the KdV and Toda equations with steplike finite-gap backgrounds	p10
09.45–10.30	Spyros Kamvissis, University of Crete Stability of the periodic Toda lattice	p15
10.30–11.00	Coffee break	
<b>Session 12</b>	Chair: Gerald Teschl, University of Vienna	
11.00–11.45	Pierre Van Moerbeke, UCL Louvain Non-intersecting Brownian motions and integrable systems	p??
11.45–12.30	Fritz Gesztesy, UMC Columbia On a generalization of the spectral problem underlying the Camassa–Holm hierarchy	p12
12.30–14.00	Lunch break	
<b>Session 13</b>	Chair: Tamara Grava, SISSA Trieste	
14.30–14.55	Dong Wang, University of Michigan Hermitian matrix model with spiked external source: universality and new discoveries	p27
14.55–15.20	Reinout Quispel, La Trobe University Discrete Integrable Systems	p26
15.20–15.45	Jiuguang Wang, Carnegie Mellon University Riemann–Hilbert problems in optimal control	p28
15.45–16.10	Viktor Enolski, Institute of Magnetism NASU On the charge three cyclic monopole	p10
16.10–16.40	Coffee break	
<b>Session 14</b>	Chair: Ira Egorova, Institute for Low Temperature Physics	
16:40–17.05	Dionyssios Mantzavinos, University of Cambridge Applications of complex analysis to initial-boundary value problems	p17
17:05–17.30	Davide Masoero, Universidade de Lisboa Some results concerning the semiclassical limit of KdV before the gradient catastrophe	p19
17.30–17.50	Hans Lundmark, Linköping University Orthogonal and biorthogonal polynomials in the theory of peakon equations	p17
17.50–18.10	Alice Mikikits-Leitner, Technical University Munich Long-time asymptotics of perturbed finite-gap Korteweg–de Vries solutions	p19
18.10–18.30	Caroline Kalla, University of Bourgogne Algebro-geometric solutions to the multi-component NLS equation: a theoretical and numerical description	p14
18.30–20.00	Poster Session	
20.00	Get-together	

# Abstracts

## Complete matchings and random matrix theory

**Jinho Baik**

University of Michigan

**Abstract.** Over the last decade or so, it has been found that the distributions that first appeared in random matrix theory describe an increasing number of objects in probability and combinatorics which do not come from matrix at all. We consider one more such an example from the so-called maximal crossing and nesting of random complete matchings. The asymptotic analysis is based on the Riemann-Hilbert problem for a Hankel determinant associated to a discrete measure which is also related to the Schur flow. This is a joint work with Bob Jenkins at the university of Michigan.

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## Long-time dynamics of step-like data for NLS+

**Anne Boutet de Monvel**

University Paris Diderot

**Abstract.** We consider the initial-value problem for the focusing nonlinear Schrödinger equation  $iq_t + q_{xx} + 2|q|^2q = 0$  on the line with “step-like” initial data:  $q(x, 0) = 0$  for  $x \leq 0$  and  $q(x, 0) = A \exp(-2iBx)$  for  $x > 0$ , where  $A > 0$  and  $B$  are real constants.

I will discuss the long-time asymptotic behavior of the solution  $q(x, t)$ . There are different qualitative asymptotic behaviors, depending on the value of  $x/t$  in the half-plane  $x \in \mathbb{R}$ ,  $t > 0$ .

Our results are obtained by the Riemann-Hilbert approach together with Deift–Zhou’s nonlinear steepest descent method.

This is joint work with Vladimir Kotlyarov and Dmitry Shepelsky.

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## Two transitions between the Pearcey and the Airy processes



**Mattia Cafasso**

Centre de Recherches Mathématiques

**Abstract.** The Pearcey and the Airy processes model the microscopic behaviour of a large number of non-intersecting Dyson brownian motions in two different regimes. In this talk I will show how, under a suitable rescaling of the variables, it is possible to approximate the Pearcey process with one/two Airy processes. This talk is based on joint works with Mark Adler, Marco Bertola and Pierre van Moerbeke.

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## Long-wave limit of periodic solutions of nonlinear wave equations

**Hongqiu CHEN**

University of Memphis

**Abstract.** In this lecture, we consider some non-linear, dispersive equations posed on the entire real line and the interest here is the relationship between two different types of solutions. One type is in Sobolev space defined on  $\mathbb{R}$  and the other is periodic, say of period  $2l$ . For the same equation, if two types of solutions are close in some sense at some time as  $l \rightarrow \infty$ , say initially, then we show that as  $l \rightarrow \infty$ , the periodic solutions converge to the solution in the Sobolev space.

Similarly, solitary-wave solution are often approximated by using periodic data. Our theory is a nice justification.

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## The hydrodynamical relevance of the Camassa–Holm and Degasperis–Procesi equations

**Adrian Constantin**

University of Vienna

**Abstract.** In recent years two nonlinear dispersive PDE's have attracted much attention due to their integrable structure. We describe how both equations arise in the modeling of the propagation of shallow water waves over a flat

bed. The equations capture stronger nonlinear effects than the classical KdV equation. In particular, they accomodate wave breaking phenomena.

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## Oscillatory Riemann-Hilbert problems and stationary phase

**Yen Do**  
Georgia Tech

**Abstract.** In this talk, I will describe some progress on asymptotics behaviors of oscillatory Riemann–Hilbert problems with non-analytic data.

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## Inverse scattering transform for the KdV and Toda equations with steplike finite-gap backgrounds

**Iryna Egorova**  
Institute for Low Temperature Physics, Kharkiv, Ukraine

**Abstract.** We present a rigorous treatment of the inverse scattering transform method (the Marchenko approach) for solving the Cauchy problem for the Korteweg - de Vries equation and for the Toda lattice with initial data, which are asymptotically close to different finite-gap backgrounds as  $x \rightarrow +\infty$  and  $x \rightarrow -\infty$  ( $n \rightarrow \pm\infty$  for the Toda lattice). Using this approach we study the long time asymptotic behavior of the solution in a neighborhood of the leading edge. We show that the solution split into solitons, the number of which grows in time.

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## On the charge three cyclic monopole

**Victor Enolski**  
Institute of Magnetism, Kiev

**Abstract.** We study  $SU(2)$  monopoles of the charge 3 in the Bogomolny-Prasad-Sommerfeld (BPS) limit with the spectral curve that respects  $C_3$ -symmetry. That is of the genus four curve taken in the form  $\eta^3 + a\eta\zeta^2 + \zeta^6 + b\zeta^3 - 1 = 0$  where  $a$  and  $b$  are two real parameters.

First we are considering the case  $a = 0$  and establish that the only curve from this family that yields BPS monopole enjoy the tetrahedrally symmetry. We introduce on this stage several new techniques making use of a factorization theorem of Fay and Accola for theta functions of unramified coverings, together with properties of the Humbert variety. The geometry leads us to a formulation purely in terms of elliptic functions.

Then we are extending this result by continuity and find (numerically) a curve in parameter  $a, b$ -plane that produces new monopole solutions. To do that a well adapted homology basis is presented enabling the theta functions and monopole data constructed by initial genus four curve to be given in genus two data. The Richelot correspondence and generalize arithmetic-geometric mean is used to solve this genus two curve.

We also expose the reconstruction of the monopole fields that is based on the Nahm Ansatz and theta-functional solutions to the Nahm equations.

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## A unified and effective method for integrable non-linear PDEs

Athanasios S. Fokas

Cambridge

**Abstract.** A new method for analyzing boundary value problems (BVPs) extending ideas of the inverse scattering transform method, was introduced in [1], [2] and further developed by several authors, in particular by G. Biondini, J. Bona, A. Boutet de Monvel, B. Deconinck, A.R. Its, V. Kotlyarov, J. Lenells, D. Mantzavinos, B. Pelloni, D. Shepelsky and L.Y. Sung. Recent unexpected developments regarding the implementation of this method to evolution PDEs in the half-line will be reviewed, which suggest the following: (i) For linearizable BVPs the new method is as effective as the usual inverse scattering transform method; this included PDEs with a third order derivative, for which the alternative approach of extension from the half-line to the full line, fails. (ii) For general (non-linearizable) BVPs, it yields the solution via a matrix Riemann–Hilbert problem, whose jump matrix can be characterized via a well defined nonlinear equation, in terms of the given initial and boundary conditions. (iii) For general BVPs, with either decaying, or  $t$ -periodic boundary conditions, it yields effective long time asymptotic formulae.

In addition, the  $x$ -periodic initial value problem with arbitrary, as opposed to “finite gap”, initial conditions can be incorporated in the new method.

## References

- [1] A.S. Fokas, *A Unified Transform Method for Solving Linear and Certain Nonlinear PDEs*, Proc. R. Soc. Lond. A **453**, 1411–1443 (1997).
- [2] A.S. Fokas, *On the Integrability of Linear and Nonlinear PDEs*, J. Math. Phys. **41**, 4188–4237 (2000).

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## On a generalization of the spectral problem underlying the Camassa–Holm hierarchy

**Fritz Gesztesy**  
UMC Columbia

**Abstract.** Using a Birman–Schwinger-type operator approach, we study generalizations of the left-definite spectral problem underlying the Camassa–Holm hierarchy. We re-derive a number of known results, including basic Floquet theoretic facts, considerably relaxing the conditions on the coefficients (permitting certain measures and distributions).

We also illustrate the manner in which the Birman–Schwinger-type approach is most natural in the sense that it yields necessary and sufficient conditions for certain relative boundedness and relative compactness results.

This is based on joint work with Rudi Weikard (UAB, Alabama, USA).

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## Universality in Hamiltonian PDEs

**Tamara Grava**  
SISSA Trieste

**Abstract.** We consider Hamiltonian PDEs and study the behaviour of solutions near critical points. Such behaviour is locally independent from the initial data and it is described by special solutions of a Painleve equations.

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## A Lipschitz metric for the Camassa–Holm equation

**Katrin Grunert**

University of Vienna

**Abstract.** When studying the Cauchy problem for the Camassa–Holm equation the question of how to continue solutions beyond wave breaking is of special interest. In the case of conservative solutions which preserve the energy for almost all times, we present how to study the stability of such solutions by deriving a Lipschitz metric. This talk is based on joint work with Helge Holden and Xavier Raynaud.

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## Initial-boundary problems for integrable nonlinear PDEs

**Iasonas Hitzazis**

University of Patras

**Abstract.** We study initial-boundary value problems (IBVPs) for certain nonlinear partial differential equations (PDEs) integrable by the so-called inverse scattering method (ISM). First applied in the middle 60's upon the initial value problem (IVP) for the celebrated Korteweg–de Vries (KdV) equation, the ISM has since been developed into a powerful technique for studying the IVP for nonlinear PDEs that admit a so-called Lax pair formulation (integrable PDEs). A generalization of the ISM, that renders it applicable upon initial-boundary value problems (IBVPs) as well, has recently been developed by A.S.Fokas and his collaborators. First applied upon the Goursat IVP for the Ernst equation, this so-called Unified Transform Method is based on a simultaneous spectral analysis of the Lax pair associated to the given integrable PDE. Alike the classical ISM, this generalized method reduces the IBVP under consideration to a Riemann–Hilbert or a DBAR problem in the spectral plane. This makes it possible to further studying the asymptotic properties of the solutions of the given IBVP by applying an appropriate nonlinear steepest descent method. In this presentation we illustrate the application of the Unified Transform Method upon certain IBVPs. In particular, we mainly address the IBVP for the KdV equation posed on a finite interval of the spatial variable. Other IBVPs will also briefly be addressed, including: (i) Moving IBVPs for integrable two-dimensional PDEs (KdV and mKdV equations), and (ii) IBVPs for multidimensional integrable PDEs, like the Davey–Stewartson equation.

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## On the nonlinear variational wave equation

**Helge Holden**

Norwegian University of Science and Technology

**Abstract.** We prove existence of a global semigroup of conservative solutions of the nonlinear variational wave equation  $u_{tt} - c(u)(c(u)u_x)_x = 0$ . This equation shares many of the peculiarities of the Hunter–Saxton and the Camassa–Holm equations. In particular, the equation possesses two distinct classes of solutions denoted conservative and dissipative. In order to solve the Cauchy problem it is necessary to augment the equation properly in order to obtain a unique solution. In this talk we describe how this is done for conservative solutions. The equation was derived by Saxton as a model for liquid crystals. The talk is based on joint work with Xavier Raynaud.

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## Algebro-geometric solutions to the multi-component NLS equation: a theoretical and numerical description

**Caroline Kalla**

University of Bourgogne, France

**Abstract.** It is well known that almost periodic solutions of certain integrable systems can be given in terms of multi-dimensional theta functions associated to compact Riemann surfaces. We present such solutions for the multi-component nonlinear Schrödinger equation (n-NLS) using a method based on Fay’s famous trisecant identity, relating the theta function and its partial derivatives. A degenerated version of the Fay identity leads to algebro-geometric solutions of the n-NLS equation. The underlying Riemann surface has to satisfy reality properties. The solutions are visualized for hyperelliptic and for general Riemann surfaces. Degenerating the Riemann surface into a genus zero surface, we obtain solutions like solitons, breathers and rational breathers, well known in the scalar case.

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## Stability of the periodic Toda lattice

**Spyros Kamvissis**

University of Crete

**Abstract.** We consider the question of the asymptotic stability of periodic (and slightly more generally of algebro-geometric finite-gap) solutions of the doubly infinite Toda lattice under a short-range perturbation. We prove that the perturbed lattice asymptotically approaches a modulated lattice instead. Our method relies on the equivalence of the inverse spectral problem to a vector Riemann–Hilbert problem defined on the hyperelliptic curve and generalizes the so-called nonlinear stationary phase/steepest descent method for Riemann–Hilbert problem deformations to Riemann surfaces.

This is joint work with Gerald Teschl.

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## Harrison cohomology and integrable systems for the hyperelliptic subsets in Birkhoff strata of Sato Grassmannian

**Boris Konopelchenko**

University of Salento, Lecce, Italy

**Abstract.** Local properties of the families  $W$  of algebraic varieties in Birkhoff strata of the Sato Grassmannian containing hyperelliptic curves of genus  $g$  are discussed. It is shown that the tangent spaces  $T$  for  $W$  are isomorphic to linear spaces of 2-coboundaries. Particular subsets in  $W$  are described by the integrable dispersionless coupled KdV hierarchies which provide us with the special class of Harrison’s 2-cocycles and 2-coboundaries. In particular, the dispersionless NLS equation describes deformations of an elliptic curves with the fixed point and gives a class of dNLS 2-cocycles and 2-coboundaries. It is shown that the blowups of such 2-cocycles and 2-coboundaries and gradient catastrophes for associated integrable systems are interrelated.

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## Long-time asymptotics for the Camassa–Holm equation

**Aleksey Kostenko**

Institute of Applied Mathematics and Mechanics NASU

**Abstract.** We apply the method of nonlinear steepest descent to compute the long-time asymptotics of the Camassa–Holm equation for decaying initial data. This is based on joint work with Anne Boutet de Monvel, Dmitry Shepelsky, and Gerald Teschl.

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## Multiple orthogonal polynomials in the normal matrix model

**Arno Kuijlaars**

Katholieke Universiteit Leuven

**Abstract.** The normal matrix model is a random matrix model defined on complex matrices. The eigenvalues in this model fill a two-dimensional region in the complex plane as the size of the matrices tends to infinity. Orthogonal polynomials with respect to a planar measure are a main tool in the analysis.

In many interesting cases, however, the orthogonality is not well-defined, since the integrals that define the orthogonality are divergent. I will present a way to redefine the orthogonality in terms of a well-defined Hermitian form. This reformulation allows for a Riemann–Hilbert characterization as multiple orthogonal polynomials. For the special case of a cubic potential it is possible to do a complete steepest descent analysis on the Riemann–Hilbert problem, which leads to strong asymptotics of the multiple orthogonal polynomials, and in particular to the two-dimensional domain where the eigenvalues are supposed to accumulate.

This is joint work with Pavel Bleher (Indianapolis).

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## Boundary value problems for the Ernst equation

**Jonatan Lenells**

Baylor University

**Abstract.** For stationary and axisymmetric spacetimes, the vacuum Einstein equations reduce to a single integrable equation in two dimensions—the celebrated Ernst equation. Using the new method introduced by Fokas, we first



recover two of the most famous solutions of this equation, the Kerr black hole and the Neugebauer–Meinel disk, as solutions of linearizable boundary value problems. We then present new exact solutions of a class of boundary value problems corresponding to a disk rotating around a central black hole. The new solutions are given explicitly in terms of theta functions on a family of Riemann surfaces of genus four.

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## Orthogonal and biorthogonal polynomials in the theory of peakon equations

**Hans Lundmark**  
Linköping University

**Abstract.** The Camassa–Holm shallow water equation is well-known for being an integrable PDE which admits non-smooth soliton solutions, usually called peakons (peaked solitons); these waves have corners where the derivative is undefined. The general  $n$ -peakon solution is given by explicit formulas which can be expressed in terms of polynomials orthogonal with respect to the spectral measure of an associated vibrating string problem. More recently, other integrable PDEs with peakon solutions have been discovered, the most widely known of these being the Degasperis–Procesi equations. Whereas the Camassa–Holm peakons are closely related to very classical mathematics (orthogonal polynomials, Padé approximation, Stieltjes continued fractions, selfadjoint Sturm–Liouville problems), the newer peakon equations have turned out to be a rich source of interesting generalizations of these concepts. For these equations, the  $n$ -peakon solution can be expressed in terms of polynomials which are biorthogonal with respect to a bilinear form involving the so-called Cauchy kernel and a spectral measure coming from an associated “cubic string” (a certain third-order non-selfadjoint boundary value problem). The talk will give an overview of how all this works, and also touch upon some recent developments.

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## Applications of complex analysis to initial-boundary value problems

**Dionyssios Mantzavinos**  
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**Abstract.** Perhaps one of the most important developments in the area of PDEs took place back in 1997, when Fokas introduced a new method for the analysis of initial-boundary value (ibv) problems for linear and integrable nonlinear PDEs on the half-line [1]. This new approach was subsequently applied to more complicated domains (see e.g. [2]-[6]), while more recently, it was extended to higher dimensions via the solution of the ibv problem for the Davey-Stewartson equation on the half-plane [7]. In this talk, we present the implementation of the new method in 2+1 dimensions (2 spatial and 1 temporal) in the case of the Kadomtsev-Petviashvili II equation, which is the two dimensional analogue of the celebrated Korteweg-de Vries equation. In particular, through the simultaneous spectral analysis of a suitable Lax pair, we are able to express the corresponding eigenfunctions (i) with respect to the physical variables via a nonlocal RH problem, and (ii) with respect to the spectral variables via a  $\bar{d}$ -bar problem. By comparing the two expressions, we obtain an integral representation for the solution of the ibv problem, involving appropriate transforms of the initial and the boundary values. A so-called global relation is also produced, relating the aforementioned transforms. The elimination of the unknown boundary values from the integral representation remains an open problem.

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## Some results concerning the semiclassical limit of KdV before the gradient catastrophe

**Davide Masoero**  
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**Abstract.** We consider the KdV equation in the semiclassical limit, namely the following Cauchy problem

$$u_t = uu_x + \epsilon u_{xxx}, \quad u(t = 0; \epsilon) = \varphi(x) \in H^s(\mathbb{R}), s \geq 3.$$

If  $\epsilon = 0$  the solution of the Cauchy problem develops a singularity at a time  $t_c > 0$ .

We show that if  $T < t_c$  and  $s \geq 3N + 3$ , the solution of the Cauchy problem is continuous and  $N$ -times differentiable with respect to  $\epsilon$ . Moreover, we compute explicitly the first derivatives at  $\epsilon = 0$ .

We then generalise the result to some deformations of the KdV equation.

This is a joint work with A. Raimondo, SISSA.

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## On the spatial asymptotics of solutions of the Ablowitz–Ladik hierarchy

**Johanna Michor**  
University of Vienna

**Abstract.** We show that for decaying solutions of the Ablowitz–Ladik system (an integrable discretization of the AKNS-ZS system), the leading asymptotic term is time independent. In addition, two arbitrary bounded solutions of the Ablowitz–Ladik system which are asymptotically close at the initial time stay close. We briefly mention the corresponding results for the associated hierarchy.

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## Long-time asymptotics of perturbed finite-gap Korteweg–de Vries solutions

**Alice Mikikits-Leitner**  
Technical University Munich

**Abstract.** In this talk we will consider one of the most famous examples of a completely integrable nonlinear wave equation, the Korteweg–de Vries (KdV) equation. The goal will be to investigate the long-time asymptotic behavior of solutions of the KdV equation which are short-range perturbations of quasi-periodic finite-gap KdV solutions. It is well-known that in the classical case with constant background in the limit for long times the following picture appears: the perturbed solution splits up into a number of solitons (solitary waves) generated by the eigenvalues of the Lax operator. Apart from that there exists a decaying radiation part corresponding to the continuous spectrum. In other words, the solitons constitute the stable part of the solution. We will investigate the case where the constant background is replaced by a quasi-periodic one. One possibility for rigorously studying this question is provided by applying the method of nonlinear steepest descent for oscillatory Riemann–Hilbert problems.

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## Fluxon condensates and the semiclassical sine-Gordon equation

Peter D. Miller

University of Michigan

**Abstract.** The semiclassically scaled sine-Gordon equation is considered with (relatively) slowly varying initial data. Considered as functions of  $x$ , the initial conditions parametrize a curve in the phase portrait of the simple pendulum. We study the solution of the initial-value problem and show that away from certain critical points  $x = x_{\text{crit}}$  where the initial data curve crosses the pendulum separatrix the solution is asymptotically described by modulated elliptic functions characterizing superluminal pulse trains of either librational (kink-antikink) type or rotational (kink-kink) type. On the other hand, near critical points  $x_{\text{crit}}$ , the behavior is far more subtle but universal (independent of details of initial conditions), being described by a specific network of kinks, antikinks, and degenerate grazing collisions of kinks spatiotemporally organized according to an infinite hierarchy of rational solutions of the Painlevé-II system. This is joint work with Robert Buckingham.

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## Long/short-time behavior of the solution to the mKdV equation with the step-like initial data

## Alexander Minakov

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**Abstract.** We consider the initial value problem for the modified Korteweg–de Vries equation:

$$\begin{aligned} q_t + 6q^2q_x + q_{xxx} &= 0, & x, t \in \mathbb{R} \\ q(x, 0) &= c_l, & x < 0, \\ &= c_r, & x > 0. \end{aligned}$$

Arbitrary real numbers  $c_l, c_r$  are positive and  $c_l > c_r$ . The goal of this report is to present the asymptotic behavior of the solution of the initial value problem as  $t \rightarrow \mp\infty$  and  $t \rightarrow +0$ . The case  $t \rightarrow -\infty$  corresponds to the long-time dynamics of the rarefaction wave.

**Theorem 1** [1] *For  $t \rightarrow -\infty$  the solution of the initial value problem takes the form*

$$q(x, t) = \begin{cases} c_l + O(e^{-Ct}), & -\infty < x < 6c_l^2t, \\ \sqrt{\frac{x}{6t}} + O(t^{-1/2}), & 6c_l^2t < x < 6c_r^2t, \\ c_r + O(t^{-1/2}), & x > 6c_r^2t. \end{cases}$$

where  $C > 0$  is some positive number.

In [2] the second term of the asymptotics is obtained in the form of the self-similar vanishing (as  $O(t^{-1/2})$ ) wave when  $x > 6c_r^2t$  and  $t \rightarrow -\infty$ .

The case  $t \rightarrow \infty$  corresponds to the long-time dynamics of the compressive wave, which was studied in the case  $c = c_l > c_r = 0$ .

**Theorem 2** [3] *Let  $t \rightarrow \infty$ . Then the solution of initial value problem takes the form of a modulated elliptic wave:*

$$q(x, t) = \begin{cases} c, & -\infty < x < -6c^2t; \\ (c + d(\xi)) \operatorname{dn} \left( U(t, \xi) \middle| m(\xi) \right) + O(t^{-1/2}), & -6c^2t < x < 4c^2t; \\ 0, & 4c^2t < x < \infty, \end{cases}$$

where  $m(\xi) = 1 - \frac{d(\xi)}{c} \in (0, 1)$  is a modulo of Jacobian elliptic function, and function  $d = d(\xi)$ , being monotone and continuous, satisfies the equation:

$$d^2 = c^2 + 2\xi - \int_0^1 \lambda^2 d^2 \sqrt{\frac{1 - \lambda^2}{c^2 - \lambda^2 d^2}} d\lambda \left( \int_0^1 \sqrt{\frac{1 - \lambda^2}{c^2 - \lambda^2 d^2}} d\lambda \right)^{-1}, \quad \xi = x/12t.$$

Function  $U(t, \xi)$  has the form  $U(t, \xi) = K[m(\xi)] (U_0(\xi) + tU_1(\xi))$ , where

$$K[m] = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - m \sin^2(\theta)}},$$

$U_1(\xi), U_0(\xi)$  are some known functions.

We have also studied the asymptotic behavior of the solution as  $t \rightarrow 0$ :

**Theorem 3** [4] *For sufficiently small  $t > 0$  and any positive  $x_0$  solution  $q(x, t)$  is smooth in  $x$  and  $t$  and has the following asymptotic behavior (as  $t \rightarrow +0$ ):*

$$q(x, t) = \begin{cases} c \int_{\tau}^{\infty} \text{Ai}(s) ds + O(t^{1/3}), & \tau = \frac{x}{(3t)^{1/3}} > -x_0, \\ c + O(t^{1/3}), & x < -6c^2t, \end{cases}$$

where  $\text{Ai}(s)$  is known Airy function.

This formula and asymptotic formulae for the Airy function as  $\tau \rightarrow \pm\infty$  ( $t \rightarrow +0$ ) show that the Gibbs-type phenomenon occurs at the point  $x = 0$ .

This is based on joint work with Vladimir Kotlyarov.

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## Thermodynamic phase transitions and shock singularities

Antonio Moro  
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**Abstract.** We show that under rather general assumptions on the functional form of the state functions, the balance equation for a gas in thermodynamic equilibrium is equivalent to a system of nonlinear equations of hydrodynamic type. This set of equations is integrable via the method of the characteristics

and provides the equation of state for the gas. The critical point of gradient catastrophe identifies the phase transition.

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## Boundary value problems for a nonlinear elliptic PDE: the elliptic sine-Gordon equation

**Beatrice Pelloni**

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**Abstract.** I will present recent work on the analysis of an elliptic integrable model, the so-called elliptic sine-Gordon equation. The analysis is based on the implementation of the method introduced by Fokas for the study of boundary value problem for integral PDES in two independent variables. Because of the elliptic nature of the problem, this implementation presents several novel features. In particular, I will discuss the Dirichlet to Neumann map for the case of boundary value problems posed in a half plane. I will also present a specific case of linearisable boundary conditions for the problem posed in a semistrip.

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## The periodic Ablowitz–Ladik equation and Floquet CMV matrices

**Irina Nenciu**

University of Illinois at Chicago

**Abstract.** We consider the periodic defocusing Ablowitz–Ladik equation and show that it can be expressed as an isospectral deformation of Floquet CMV matrices. We then introduce a Poisson Lie group whose underlying group is a loop group and show that the set of Floquet CMV matrices is a Coxeter dressing orbit of this Poisson Lie group. By using the group-theoretic framework, we establish the Liouville integrability of the equation by constructing action-angle variables, we also solve the Hamiltonian equations generated by the commuting flows via Riemann-Hilbert factorization problems. This is joint work with L.-C. Li.

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## The KdV equation, the Titchmarsh-Weyl $m$ -function and Hankel operators

Alexei Rybkin

University of Alaska Fairbanks

**Abstract.** This talk is concerned with linking together the theory of Hankel operators and completely integrable systems. On the prototypical example of the Cauchy problem for the KdV equation we demonstrate the power of the language of Hankel operators which symbols are conveniently represented in terms of the Titchmarsh-Weyl  $m$ -functions associated with the initial data. Among others, this approach yields some new wellposedness results for the initial value problem for the KdV equation.

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## Weyl theory, inverse spectral transform, and initial-boundary value problems for integrable equations

Alexander Sakhnovich

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**Abstract.** Inverse spectral transform is a fruitful approach to initial-boundary value problems. We present several examples as well as new results on the Weyl theory and inverse spectral transform for Dirac system with rectangular potential.

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## The short-wave model for the Camassa–Holm equation: the Riemann–Hilbert approach

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**Abstract.** We present the inverse scattering transform approach to the Cauchy problem on the line for the short-wave model for the Camassa–Holm equation

$$u_{txx} - 2u_x + 2u_x u_{xx} + uu_{xxx} = 0$$



in the form of the associated Riemann–Hilbert problem. This approach allows us to give a representation of the classical (smooth) solutions, describe their asymptotics as  $t \rightarrow \infty$ , and describe cuspons — non-smooth soliton solutions with infinite slope at the crest.

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## Random matrices: finite rank deformations and fluctuations of matrix entries of regular functions

Alexander Soshnikov

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**Abstract.** In the first part of the talk I will discuss the results about fluctuation of the outliers in the spectrum of finite rank deformations of Wigner random matrices. This question is related to the limiting distribution of the resolvent entries of standard Wigner matrices. The second part of the talk will be devoted to the results about the fluctuation of the matrix entries of regular functions of Wigner and sample covariance random matrices.

The results are joint with Sean O’Rourke, Alessandro Pizzo, and David Renfrew (preprints [arXiv:1103.1170](https://arxiv.org/abs/1103.1170), [arXiv:1103.3731](https://arxiv.org/abs/1103.3731), and [arXiv:1104.1663](https://arxiv.org/abs/1104.1663)).

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## Burgers turbulence, kinetic theory of shock clustering, and complete integrability

Ravi Srinivasan

University of Texas at Austin

**Abstract.** A remarkable stochastic coalescence model arises from considering shock statistics in scalar conservation laws with Markov initial data. While rooted in the study of Burgers turbulence, the model has deep connections to kinetic theory, random matrices, and statistics. The evolution takes the form of a Lax equation, which admits some rather intriguing exact solutions. More recently, it has been shown that this flow on ‘Markov groups’ is a completely integrable Hamiltonian system and is a natural analogue of the N-wave model from nonlinear optics. We will provide a broad overview of these results and their various connections. This talk consists of joint work with Govind Menon (Brown University, USA).

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## On an inverse spectral problem associated to a two-potential generalization of the Degasperis–Procesi equation

Jacek Szmigielski

University of Saskatchewan, Saskatoon, Canada

**Abstract.** In this talk I will show a construction of peakon solutions to the integrable system of coupled nonlinear equations depending on two potentials first introduced by X. Geng and B. Xue. This system generalizes in a symmetric way Degasperis–Procesi (DP) and V. Novikov equations (VN). The construction of solutions in this class relies on the use of Cauchy biorthogonal polynomials and inverse spectral methods both of which will be explained during the talk. One of the novel features of our solution is that one uses two distinct Lax pairs to do the construction and to carry out the inverse problem. The direct spectral problem is non-selfadjoint but can be studied using ideas related to the concept of Gantmacher–Krein oscillatory kernels. The inverse spectral problem is solved using certain Hermite–Padé approximations and Cauchy biorthogonal polynomials which were introduced in connection with the inverse problem pertinent to the DP equation. A comparison will be made between this construction and preceding results for DP and VN. This is joint work with H. Lundmark.

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## Discrete integrable systems

Reinout Quispel

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**Abstract.** Integrable ordinary differential equations (ODEs) have been studied intensively since the time of Kepler and Newton, integrable partial differential (PDEs) equations since the discovery of solitons in the Korteweg–de Vries equation by Kruskal and collaborators. Both integrable ODEs as well as PDEs are of great importance in mathematics as well as in applications (e.g. as a starting point for perturbation theory). The question thus arises how to generalize this theory and its applications from the continuous case (integrable differential equations) to the discrete case (integrable difference equations), i.e. when all independent variables are discrete. This question is important both from the point

of view of mathematics (where the discrete case may be viewed as a deformation of the continuous case), as well as from the point of view of applications. The history of discrete integrable systems goes back to seminal papers in the 1970s by Ablowitz and Ladik and by Hirota. In the early 1980s, systematic methods for the construction of integrable nonlinear partial difference equations (P $\Delta$ Es) were found, e.g. through the representation theory of infinite-dimensional Lie algebra, or in the work of Quispel, Nijhoff and co-workers on singular integral equations and connections with Bäcklund transformations. Integrable ordinary difference equations (O $\Delta$ Es) were first extensively studied by Quispel, Roberts, and Thompson (QRT) in the second-order case, and by Papageorgiou, Nijhoff and Capel, and by Quispel et al. in the higher-order case. As is not uncommon when one studies deformations of a theory, the discrete case exhibits both similarities to, as well as differences from, the continuous case.

Discrete integrable systems may be (in a sense) more fundamental than continuous ones. This is one reason for their significance. Not only does e.g. the continuous KdV equation arise in the continuum limit from the discrete KdV equation, but in fact, the entire infinite hierarchy of continuous KdV equations arises in the limit from the single discrete KdV equation! This was shown in an important, but little-known, paper by Wiersma and Capel using vertex operator methods. (Similarly for other integrable P $\Delta$ Es.) Related to this is the fact that discrete integrable P $\Delta$ Es exhibit a much higher symmetry than their continuous counterparts, this symmetry being broken in the continuum limit. While this symmetry of integrable P $\Delta$ Es and of their Lax pairs under permutation of the (discrete) spatial independent variables has been very explicit since e.g. our early work, a much fuller understanding has been obtained more recently in terms of so-called multi-dimensional consistency.

In this talk, we will present some recent results from this exciting area.

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## **Hermitian matrix model with spiked external source: universality and new discoveries**

**Dong Wang**  
University of Michigan

**Abstract.** In this talk I will discuss the limiting distribution of top eigenvalues in the Hermitian matrix model with general potential and spiked external source, i.e., the external source is of finite rank. For a large class of potentials, the phase transition phenomenon that was observed for Gaussian and Laguerre potentials is proved to be universal. However, new types of transitions are discovered for other classes of potentials.

This is joint work with Jinho Baik.

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## Riemann–Hilbert problems in optimal control

Jiuguang Wang

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### Abstract.

We present a Riemann–Hilbert formulation of optimal control theory as applied to problems in robotics. Continuous time optimal control has traditionally been based on resolving necessary and sufficient conditions for optimality, most notably the Hamilton–Jacobi–Bellman (HJB) equation, Pontryagin’s minimum principle, and their numerical approximations. It is unusual to derive exact analytical solutions to these problems since it typically involves solving a set of partial differential equations that do not, in general, yield smooth solutions. Instead, numerical tools such as dynamic programming are used to obtain a discretized globally optimal control policy. Other trajectory optimization tools based on nonlinear programming can be used to obtain approximate solutions that can be verified to be locally optimal. While these numerical approximations have been proven to be effective in solving real-world problems, especially given complex physical constraints, they suffer from the *curse of dimensionality* and require extensive computational resources to pre-compute control policies offline. When available, analytical solutions offer significant computational advantages over open-loop trajectory optimizers and can be easily implemented in a closed-loop setting for high-dimensional systems. In this work, we formulate a class of optimal control problems in the complex plane and show that the resulting Riemann–Hilbert problem can be solved with calculus of variations. We show how closed-form, analytical solutions can be obtained from this formulation and the use of these results in robotics applications.

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