CHAPTER I

ON EXPLICATION

After a brief indication of the problems to be dealt with in this book—the problems of degree of confirmation, induction, and probability (§ 1)—the remainder of this chapter contains a discussion of some general questions of a methodological nature. By an explication we understand the transformation of an inexact, prescientific concept, the explicandum, into an exact concept, the explicatum (§ 2). The explicatum must fulfil the requirements of similarity to the explicandum, exactness, fruitfulness, and simplicity (§ 3). Three kinds of concepts are distinguished: classificatory (e.g., Warm), comparative (e.g., Warmer), and quantitative concepts (e.g., Temperature) (§ 4). The role of comparative and quantitative concepts as explicata is discussed (§ 5). The axiomatic method is briefly characterized, and the distinction between its two phases, formalization and interpretation, is especially emphasized (§ 6). In this chapter the methodological questions are discussed in a general way, without reference to the specific problems of this book. Only in later chapters will the results of these preliminary explanations be applied in the discussions concerning confirmation and probability.

§ 1. Introduction: Our Problems

A brief, preliminary indication is given of the tasks which this book will try to solve: a clarification of (1) degree of confirmation, (2) induction, (3) probability.

The chief tasks of this book will be:

1) a clarification and, if possible, a definition of the concept of *degree of confirmation;*
2) a clarification of the logical nature of *induction* and, if possible, a construction of a system of *inductive logic;*
3) a clarification of the concept of *probability*.

At the present only a few preliminary explanations of these problems will be given.

1. When scientists speak about a scientific law or a theory, or also a singular statement, for example, a prediction, on the one hand, and certain observational data or experimental results, on the other, they often state a relation between those items in forms like these:

a. 'This experiment again confirms the theory T' (or: '. . . supplies new evidence for . . .').

b. 'The quantum theory is confirmed to a considerably higher degree by the experimental data known today than by those available twenty years ago' (or: '. . . is supported more strongly by . . .').

The concepts of confirming evidence or degree of confirmation used in statements of this kind are usually sufficiently well understood for simple, practical purposes, but they are hardly ever precisely explained. It will be one of the chief tasks of this book to make concepts of this kind precise and to furnish a theory of the logical relations between any hypothesis and any piece of knowledge that might be regarded as confirming evidence for the hypothesis.

2. The problem of induction in the widest sense—concerning a hypothesis of any, not necessarily universal form—is essentially the same as the problem of the logical relation between a hypothesis and some confirming evidence for it. Thus, by laying down a definition for the concept of degree of confirmation and constructing a logical theory based upon this concept, we shall furnish a system of inductive logic. While deductive logic may be regarded as the theory based upon the concept of logical consequence or deducibility, inductive logic is the theory based upon what might be called the degree of inducibility, that is, the degree of confirmation.

3. The problem of probability is likewise closely related to that of induction. This has often been observed, at least with respect to one of the various conceptions of probability which we find in the historical development (sometimes called inductive probability). We shall try to show that we have to distinguish chiefly two concepts of probability; the one is defined in terms of frequency and is applied empirically, the other is a logical concept and is the same as degree of confirmation. It will be shown that both are important for the method of science, and thus the controversy between the two "conceptions" of probability will be dissolved.

Thus we see that one or several of the problems which we intend to approach have the following character. There is a certain term ('confirming evidence', 'degree of confirmation', 'probability') which is used in everyday language and by scientists without being exactly defined, and we try to make the use of these terms more precise or, as we shall say, to give an *explication* for them. The task of explication is of very general importance for the construction of concepts. Therefore we shall devote the remainder of this chapter (§§ 2–6) to a discussion of the general nature of the method of explication and only in the next chapter (§ 8) return to our specific problems of confirmation and probability.

§ 2. On the Clarification of an Explicandum

> By the procedure of *explication* we mean the transformation of an inexact, prescientific concept, the *explicandum*, into a new exact concept, the *explicatum*. Although the explicandum cannot be given in exact terms, it should be made as clear as possible by informal explanations and examples.

The task of **explication** consists in transforming a given more or less inexact concept into an exact one or, rather, in replacing the first by the second. We call the given concept (or the term used for it) the **explicandum**, and the exact concept proposed to take the place of the first (or the term proposed for it) the **explicatum**. The explicandum may belong to everyday language or to a previous stage in the development of scientific language. The explicatum must be given by explicit rules for its use, for example, by a definition which incorporates it into a well-constructed system of scientific either logicomathematical or empirical concepts.

> The term 'explicatum' has been suggested by the following two usages. Kant calls a judgment explicative if the predicate is obtained by analysis of the subject. Husserl, in speaking about the synthesis of identification between a confused, nonarticulated sense and a subsequently intended distinct, articulated sense, calls the latter the 'Explikat' of the former. (For both uses see *Dictionary of philosophy* [1942], ed. D. Runes, p. 105). What I mean by 'explicandum' and 'explicatum' is to some extent similar to what C. H. Langford calls 'analysandum' and 'analysans': "the analysis then states an appropriate relation of equivalence between the analysandum and the analysans" ("The notion of analysis in Moore's philosophy", in *The philosophy of G. E. Moore* [1943], ed. P. A. Schilpp, pp. 321–42; see p. 323); he says that the motive of an analysis "is usually that of supplanting a relatively vague idea by a more precise one" (*ibid.*, p. 329).
> (Perhaps the form 'explicans' might be considered instead of 'explicatum'; however, I think that the analogy with the terms 'definiendum' and 'definiens' would not be useful because, if the explication consists in giving an explicit definition, then both the definiens and the definiendum in this definition express the explicatum, while the explicandum does not occur.) The procedure of explication is here understood in a wider sense than the procedures of analysis and clarification which Kant, Husserl, and Langford have in mind. The explicatum (in my sense) is in many cases the result of an analysis of the explicandum (and this has motivated my choice of the terms); in other cases, however, it deviates deliberately from the explicandum but still takes its place in some way; this will become clear by the subsequent examples.

A problem of explication is characteristically different from ordinary scientific (logical or empirical) problems, where both the datum and the solution are, under favorable conditions, formulated in exact terms (for example, 'What is the product of 3 and 5?', 'What happens when an electric current goes through water?'). In a problem of explication the datum,

viz., the explicandum, is not given in exact terms; if it were, no explication would be necessary. Since the datum is inexact, the problem itself is not stated in exact terms; and yet we are asked to give an exact solution. This is one of the puzzling peculiarities of explication. It follows that, if a solution for a problem of explication is proposed, we cannot decide in an exact way whether it is right or wrong. Strictly speaking, the question whether the solution is right or wrong makes no good sense because there is no clear-cut answer. The question should rather be whether the proposed solution is satisfactory, whether it is more satisfactory than another one, and the like. What is meant by these questions will soon be made clearer.

Before we turn to the chief question, viz., what are the requirements for a satisfactory solution of a problem of explication, that is to say, for a satisfactory explicatum, let us look somewhat more at the way in which the problem is to be stated, that is, how the explicandum is to be given. There is a temptation to think that, since the explicandum cannot be given in exact terms anyway, it does not matter much how we formulate the problem. But this would be quite wrong. On the contrary, since even in the best case we cannot reach full exactness, we must, in order to prevent the discussion of the problem from becoming entirely futile, do all we can to make at least practically clear what is meant as the explicandum. What X means by a certain term in contexts of a certain kind is at least practically clear to Y if Y is able to predict correctly X's interpretation for most of the simple, ordinary cases of the use of the term in those contexts. It seems to me that, in raising problems of analysis or explication, philosophers very frequently violate this requirement. They ask questions like: 'What is causality?', 'What is life?', 'What is mind?', 'What is justice?', etc. Then they often immediately start to look for an answer without first examining the tacit assumption that the terms of the question are at least practically clear enough to serve as a basis for an investigation, for an analysis or explication. Even though the terms in question are unsystematic, inexact terms, there are means for reaching a relatively good mutual understanding as to their intended meaning. An indication of the meaning with the help of some examples for its intended use and other examples for uses not now intended can help the understanding. An informal explanation in general terms may be added. All explanations of this kind serve only to make clear what is meant as the explicandum; they do not yet supply an explication, say, a definition of the explicatum; they belong still to the formulation of the problem, not yet to the construction of an answer. (Examples. 1. I might say, for example: "I mean by the explicandum 'salt', not its wide sense which it has in chemistry but its nar-

row sense in which it is used in the household language". This explanation is not yet an explication; the latter may be given, for instance, by the compound expression 'sodium chloride' or the synonymous symbol 'NaCl' of the language of chemistry. 2. "I am looking for an explication of the term 'true', not as used in phrases like 'a true democracy', 'a true friend', etc., but as used in everyday life, in legal proceedings, in logic, and in science, in about the sense of 'correct', 'accurate', 'veridical', 'not false', 'neither error nor lie', as applied to statements, assertions, reports, stories, etc." This explanation is not yet an explication; an explication may be given by a definition within the framework of semantical concepts, for example, by Tarski's definition of 'true' in [Wahrheitsbegriff] (for abbreviated titles in square brackets see the Bibliography at the end of this volume), or by D17-1 below. By explanations of this kind the reader may obtain step by step a clearer picture of what is intended to be included and what is intended to be excluded; thus he may reach an understanding of the meaning intended which is far from perfect theoretically but may be sufficient for the practical purposes of a discussion of possible explications.

§ 3. Requirements for an Explicatum

A concept must fulfil the following requirements in order to be an adequate explicatum for a given explicandum: (1) similarity to the explicandum, (2) exactness, (3) fruitfulness, (4) simplicity.

Suppose we wish to explicate a certain prescientific concept, which has been sufficiently clarified by examples and explanations as just discussed. What is the explication of this concept intended to achieve? To say that the given prescientific concept is to be transformed into an exact one means, of course, that an exact concept corresponding to the given concept is to be introduced. What kind of correspondence is required here between the first concept, the explicandum, and the second, the explicatum?

Since the explicandum is more or less vague and certainly more so than the explicatum, it is obvious that we cannot require the correspondence between the two concepts to be a complete coincidence. But one might perhaps think that the explicatum should be as close to or as similar with the explicandum as the latter's vagueness permits. However, it is easily seen that this requirement would be too strong, that the actual procedure of scientists is often not in agreement with it, and for good reasons. Let us consider as an example the prescientific term 'fish'. In the construction of a systematic language of zoölogy, the concept Fish designated by this term has been replaced by a scientific concept designated by the same term

'fish'; let us use for the latter concept the term 'piscis' in order to avoid confusion. When we compare the explicandum Fish with the explicatum Piscis, we see that they do not even approximately coincide. The latter is much narrower than the former; many kinds of animals which were subsumed under the concept Fish, for instance, whales and seals, are excluded from the concept Piscis. [The situation is not adequately described by the statement: 'The previous belief that whales (in German even called 'Walfische') are also fish is refuted by zoölogy'. The prescientific term 'fish' was meant in about the sense of 'animal living in water'; therefore its application to whales, etc., was entirely correct. The change which zoölogists brought about in this point was not a correction in the field of factual knowledge but a change in the rules of the language; this change, it is true, was motivated by factual discoveries.] That the explicandum Fish has been replaced by the explicatum Piscis does not mean that the former term can always be replaced by the latter; because of the difference in meaning just mentioned, this is obviously not the case. The former concept has been succeeded by the latter in this sense: the former is no longer necessary in scientific talk; most of what previously was said with the former can now be said with the help of the latter (though often in a different form, not by simple replacement). It is important to recognize both the conventional and the factual components in the procedure of the zoölogists. The conventional component consists in the fact that they could have proceeded in a different way. Instead of the concept Piscis they could have chosen another concept—let us use for it the term 'piscis*'—which would likewise be exactly defined but which would be much more similar to the prescientific concept Fish by not excluding whales, seals, etc. What was their motive for not even considering a wider concept like Piscis* and instead artificially constructing the new concept Piscis far remote from any concept in the prescientific language? The reason was that they realized the fact that the concept Piscis promised to be much more fruitful than any concept more similar to Fish. A scientific concept is the more fruitful the more it can be brought into connection with other concepts on the basis of observed facts; in other words, the more it can be used for the formulation of laws. The zoölogists found that the animals to which the concept Fish applies, that is, those living in water, have by far not as many other properties in common as the animals which live in water, are cold-blooded vertebrates, and have gills throughout life. Hence the concept Piscis defined by these latter properties allows more general statements than any concept defined so as to be more similar to Fish; and this is what makes the concept Piscis more fruitful.

In addition to fruitfulness, scientists appreciate simplicity in their concepts. The simplicity of a concept may be measured, in the first place, by the simplicity of the form of its definition and, second, by the simplicity of the forms of the laws connecting it with other concepts. This property, however, is only of secondary importance. Many complicated concepts are introduced by scientists and turn out to be very useful. In general, simplicity comes into consideration only in a case where there is a question of choice among several concepts which achieve about the same and seem to be equally fruitful; if these concepts show a marked difference in the degree of simplicity, the scientist will, as a rule, prefer the simplest of them.

According to these considerations, the task of explication may be characterized as follows. If a concept is given as explicandum, the task consists in finding another concept as its explicatum which fulfils the following requirements to a sufficient degree.

1. The explicatum is to be *similar to the explicandum* in such a way that, in most cases in which the explicandum has so far been used, the explicatum can be used; however, close similarity is not required, and considerable differences are permitted.

2. The characterization of the explicatum, that is, the rules of its use (for instance, in the form of a definition), is to be given in an *exact* form, so as to introduce the explicatum into a well-connected system of scientific concepts.

3. The explicatum is to be a *fruitful* concept, that is, useful for the formulation of many universal statements (empirical laws in the case of a nonlogical concept, logical theorems in the case of a logical concept).

4. The explicatum should be as *simple* as possible; this means as simple as the more important requirements (1), (2), and (3) permit.

> Philosophers, scientists, and mathematicians make explications very frequently. But they do not often discuss explicitly the general rules which they follow implicitly. A good explicit formulation is given by Karl Menger in connection with his explication of the concept of dimension ("What is dimension?" *Amer. Math. Monthly*, 50 [1943], 2-7; see p. 5: § 3 "Criteria for a satisfactory definition" [explication, in our terminology]). He states the following requirements. The explicatum "must include all entities which are always denoted and must exclude all entities which are never denoted" by the explicandum. The explication "should extend the use of the word by dealing with objects not known or not dealt with in ordinary language. With regard to such entities, a definition [explication] cannot help being arbitrary." The explication "must yield many consequences," theorems possessing "generality and simplicity" and connecting the explicatum with concepts of other theories. See also the discussions by C. H. Langford, referred to in § 2.
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> *Terminological remarks*. 1. The word *'concept'* is used in this book as a con-

venient common designation for properties, relations, and functions. [Note that
(a) it does not refer to terms, i.e., words or phrases, but to their meanings, and
(b) it does not refer to mental occurrences of conceiving but to something ob-
jective.] For more detailed explanations see [Semantics], p. 230; [Meaning],
p. 21. 2. If I speak about an expression (e.g., a word, a phrase, a sentence, etc.)
in distinction to what is meant or designated by it, I include it in *quotation
marks*. That this distinction is necessary in order to avoid confusion has become
more and more clear in the recent development of logic and analysis of language.
3. If I want to speak about a concept (property, relation, or function) desig-
nated by a word, I sometimes use the device of *capitalizing* the word, especially
if it is not a noun (compare [Meaning], p. 17 n.). For example, I might write
'the relation Warmer'; to write instead 'the relation warmer' would look strange
and be contrary to English grammar; to write 'the relation of x being warmer
than y' would be inconvenient because of its length; the customary way of
writing 'the relation 'warmer' ' would not be quite correct, because 'warmer' is
not a relation but a word designating a relation. Similarly, I shall sometimes
write: 'the property (or concept) Fish' (instead of 'the property of being a fish');
'the property (or concept) Red' (instead of 'the property of being red' or 'the
property of redness'), and the like.

Arne Naess defines and uses a concept which seems related to our con-
cept Explicatum ("Interpretation and preciseness. I. Survey of basic con-
cepts" [Oslo Universitetets Studentkontor, 1947] [mimeographed]; this
is the first chapter of a forthcoming book). Naess defines 'the formula-
tion U is more precise than T (in the sense that U may with profit be
substituted for T)' by 'there are interpretations of T which are not inter-
pretations of U, but there are no interpretations of U which are not also
interpretations of T' (*ibid.*, p. 38). This comparative concept enables
Naess to deal with a series of consecutive "precisations" of a given con-
cept. Naess announces that a later chapter (iii) of the book will be "de-
voted to the question of how to measure degrees of ambiguity, vague-
ness, and similar properties". The comparative concept mentioned and
these quantitative concepts may prove to be effective tools for a more
penetrating analysis of explication.

§ 4. Classificatory, Comparative, and Quantitative Concepts

A *classificatory* concept (e.g., Warm) serves for classifying things into two
kinds. A *comparative* concept is a relation based on a comparison, with the sense
of 'more (in a certain respect)' (e.g., Warmer) or 'more or equal'. A *quantitative*
concept serves to describe something with the help of numerical values (e.g.,
temperature).

Among the kinds of concept used in science, three are of special im-
portance. We call them classificatory, comparative, and quantitative con-
cepts. We shall make use of this distinction in our later discussion of
confirmation and probability. In prescientific thinking classificatory con-

cepts are used most frequently. In the course of the development of science they are replaced in scientific formulations more and more by concepts of the two other kinds, although they remain always useful for the formulation of observational results. *Classificatory concepts* are those which serve for the classification of things or cases into two or a few mutually exclusive kinds. They are used, for example, when substances are divided into metals and nonmetals, and again the metals into iron, copper, silver, etc.; likewise, when animals and plants are divided into classes and further divided into orders, families, genera, and, finally, species; when the things surrounding us are described as warm or cold, big or small, hard or soft, etc., or when they are classified as houses, stones, tables, men, etc. In these examples the classificatory concepts are properties. In other cases they are relations, for example, those designated by the phrases 'x is close to y' and 'the person x is acquainted with the field of science y'. (A relation may be regarded as a property of ordered pairs.) *Quantitative concepts* (also called metrical or numerical concepts or numerical functions) are those which serve for characterizing things or events or certain of their features by the ascription of numerical values; these values are found either directly by measurement or indirectly by calculation from other values of the same or other concepts. Examples of quantitative concepts are length, length of time, velocity, volume, mass, force, temperature, electric charge, price, I.Q., infantile mortality, etc. In many cases a quantitative concept corresponds to a classificatory concept. Thus temperature corresponds to the property Warm; and the concept of a distance of less than five miles corresponds to the relation of proximity. The method of quantitative concepts and hence of measurement was first used only for physical events but later more and more in other fields also, especially in economics and psychology. Quantitative concepts are no doubt the most effective instruments in the scientific arsenal. Sometimes scientists, especially in the fields of social science and psychology, hold the view that, in cases where no way is discovered for the introduction of a quantitative concept, nothing remains but to use concepts of the simplest kind, that is, classificatory ones. Here, however, they overlook the possibility and usefulness of *comparative concepts*, which, in a sense, stand between the two other kinds. Comparative concepts (sometimes called topological or order concepts) serve for the formulation of the result of a comparison in the form of a more-less-statement without the use of numerical values. Before the scientific, quantitative concept of temperature was introduced, everyday language contained comparative concepts. Instead of merely classifying things into a few kinds

with the help of terms like 'hot', 'warm', 'luke-warm', 'cold', a more effective characterization was possible by saying that x is warmer than y (or colder, or equally warm, as the case may be).

A comparative concept is always a relation. If the underlying classificatory concept is a property (e.g., Warm), the comparative concept is a dyadic relation, that is, one with two arguments (e.g., Warmer). If the classificatory concept is a dyadic relation (e.g., the relation of x being acquainted with (the field) y), the comparative concept has, in general, four arguments (e.g., the relation of x being better acquainted with y than u with v). It is sometimes useful to regard the tetradic relation as a dyadic relation between two pairs. (We might say, for example: 'the relation of being acquainted holds for the pair x, y to a higher degree than for the pair u, v'.) Sometimes the introduction of a triadic relation is preferred to that of a tetradic relation. If we do not know how to compare the degree of Peter's knowledge in physics with Jack's knowledge in history, we might perhaps be content to use either or both of the two triadic relations expressed by the following phrases: 'x is better acquainted with (the field) y than with v', 'x is better acquainted with y than u'. The first of these two relations requires that we are able to compare the degree of Peter's knowledge in physics with that in history, which might seem problematical. The second relation involves the comparison of Peter's knowledge in physics with that of Jack; here it seems easier to invent suitable tests.

Each of the comparative concepts given above as an example has the meaning of 'more' or 'to a higher degree' with respect to a given classificatory concept. To any of those classificatory concepts (e.g., Warm), we can likewise construct a comparative concept meaning 'less' or 'to a lower degree' (e.g., Less-warm; in other words, Colder); this is the converse of the first comparative concept. In either case the comparative concept, regarded as a dyadic relation (of simple entities, pairs, etc.), has obviously the following relational properties: it is irreflexive, transitive, and (hence) asymmetric. (For definitions of these and other terms of the theory of relations see D25-2.)

In addition to the form of comparative concepts just mentioned, there is another form, less customary but often more useful. A concept of this second kind does not mean 'more' but 'more or equal' with respect to the underlying classificatory concept, in other words, 'to at least the same degree', that is, 'to the same or a higher degree' (e.g., the relation of x being at least as warm as y). Or it may mean 'less or equal' (e.g., the relation of x being less warm than y or equally as warm as y; in other words, of x being at most as warm as y). It is easily seen that a comparative con-

cept of this second kind, regarded as a dyadic relation, is reflexive and transitive but neither symmetric nor asymmetric. A comparative relation is sometimes of such a kind that, for any x and y, it holds either between x and y or between y and x (or both). In this case the relation (for example, Warmer-Or-Equally-Warm) orders its members in a kind of linear order. If, however, the condition is not fulfilled, then there are incomparable cases. Thus it might perhaps be that we find it possible to compare the scientific achievements of two persons if both work in the same field, while we do not know a way of comparing a physicist with a historian.

In everyday language the first form of comparative concept is much more customary than the second. There are many single words for those of the first form, for instance, 'above', 'beyond', 'after', etc., and especially the comparatives, for instance, 'more', 'warmer', etc., while there are hardly any single words for those of the second form. On the other hand, there is a general trend in the development of the language of science toward concepts which are wider than corresponding concepts of pre-scientific language by including extreme cases, especially cases of zero value or of identity or equality; for example, the term 'number' is now taken as including 0, 'class' as including the null class, 'velocity' as including the case of rest regarded as velocity 0, etc. With respect to comparative concepts, this trend means a development from those of the first kind to those of the second, because the latter include the boundary case of equality. One advantage of those of the second kind consists in the fact that on the basis of 'more or equal' we can define both 'equal' and 'more' ('$x = y$' can be defined by '$x \geqq y$ and $y \geqq x$'; '$x > y$' by '$x \geqq y$ and not $y \geqq x$'), while on the basis of 'more' we cannot define either 'equal' or 'more or equal'. For these reasons, when we come to a discussion of a comparative concept of confirmation (§ 8), we shall take one of the second form, as expressed by: 'h is confirmed by e to the same or a higher degree than h' by e''.

> For an analysis of comparative and quantitative concepts and an explanation of the steps to be taken in the construction of concepts of these kinds see Carnap, *Physikalische Begriffsbildung* (Karlsruhe, 1926). C. G. Hempel and P. Oppenheim have developed and improved the characterizations of the two kinds of concept and illustrated their roles in various fields of science in their book *Der Typusbegriff im Lichte der neuen Logik: Wissenschaftstheoretische Untersuchungen zur Konstitutionsforschung und Psychologie* (Leiden, 1936).

§ 5. Comparative and Quantitative Concepts as Explicata

> The role of comparative and quantitative concepts as explicata is discussed in preparation for a later discussion of comparative and quantitative concepts of confirmation.

Classificatory concepts are the simplest and least effective kind of concept. Comparative concepts are more powerful, and quantitative concepts still more; that is to say, they enable us to give a more precise description of a concrete situation and, more important, to formulate more comprehensive general laws. Therefore, the historical development of the language is often as follows: a certain feature of events observed in nature is first described with the help of a classificatory concept; later a comparative concept is used instead of or in addition to the classificatory concept; and, still later, a quantitative concept is introduced. (These three stages of development do, of course, not always occur in this temporal order.)

The situation may be illustrated with the help of the example of those concepts which have led to the quantitative concept of temperature. The state of bodies with respect to heat can be described in the simplest and crudest way with the help of classificatory concepts like Hot, Warm, and Cold (and perhaps a few more). We may imagine an early, not recorded stage of the development of our language where only these classificatory terms were available. Later, an essential refinement of language took place by the introduction of a comparative term like 'warmer'. In the case of this example, as in many others, this second step was already made in the prescientific language. Finally, the corresponding quantitative concept, that of temperature, was introduced in the construction of the scientific language.

The concept Temperature may be regarded as an explicatum for the comparative concept Warmer. The first of the requirements for explicata discussed in § 3, that of similarity or correspondence to the explicandum, means in the present case the following: The concept Temperature is to be such that, in most cases, if x is warmer than y (in the prescientific sense, based on the heat sensations of the skin), then the temperature of x is higher than that of y. Here a few remarks may be made.

(i) The requirement refers to most cases, not to all cases. It is easily seen that the requirement is fulfilled only in this restricted sense. Suppose I enter a moderately heated room twice, first coming from an overheated room and at a later time coming from the cold outside. Then it may happen that I declare the room, on the basis of my sensations, to be warmer the second time than the first, while the thermometer shows at the second time the same temperature as at the first (or even a slightly lower one). Experiences of this kind do not at all lead us to the conclusion that the concept Temperature defined with reference to the thermometer is inadequate as an explicatum for the concept Warmer. On the contrary, we have become accustomed to let the scientific concept overrule the prescientific

one in all cases of disagreement. In other words, the term 'warmer' has undergone a change of meaning. Its meaning was originally based directly on a comparison of heat sensations, but, after the acceptance of the scientific concept Temperature into our everyday language, the word 'warmer' is used in the sense of 'having a higher temperature'. Thus the experience described above is now formulated as follows: "I believed that the room was at the second time warmer than at the first, but this was an error; the room was actually not warmer; I found this out with the help of the thermometer". For this second, scientific meaning of 'warmer' we shall use in the following discussion the term 'warmer*'.

(ii) The converse of the requirement mentioned above would be this: the concept Temperature is to be such that, if x is not warmer than y (in the prescientific sense), then the temperature of x is not higher than that of y. It is important to realize that this is not required, not even "in most cases". When the difference between the temperatures of x and y is small, then, as a rule, we notice no difference in our heat sensations. This again is not taken as a reason for rejecting the concept Temperature. On the contrary, here again we have become accustomed to the new, scientific concept Warmer*, and thus we say: "x is actually warmer* than y, although we cannot feel the difference".

(iii) Thus, we have two scientific concepts corresponding to the prescientific concept Warmer. The one is the comparative concept Warmer*, the other the quantitative concept Temperature. Either of them may be regarded as an explicatum of Warmer. Both are defined with reference to the thermometer. Since the thermometer has a higher discriminating power than our heat sensations, both scientific concepts are superior to the prescientific one in allowing more precise descriptions. The procedure leading from the explicandum to either of the two explicata is as follows. At first the prescientific concept is guiding us in our choice of an explicatum (with possible exceptions, as discussed earlier). Once an explicatum is defined in a relatively simple way, we follow its guidance in cases where the prescientific concept is not sufficiently discriminative. It would be possible but highly inadvisable to define a concept Temperature in such a way that x and y are said to have the same temperature whenever our sensations do not show a difference. This concept would be in closer agreement with the explicandum than the concept Temperature actually used. But the latter has the advantage of much greater simplicity both in its definition—in other words its method of measurement—and in the laws formulated with its help.

(iv) Of the two scientific terms 'warmer*' and 'temperature', the latter

is the one important for science; the former serves merely as a convenient abbreviation for 'having a higher temperature'. The quantitative concept Temperature has proved its great fruitfulness by the fact that it occurs in many important laws. This is not always the case with quantitative concepts in science, even if they are well defined by exact rules of measurement. For instance, it has sometimes occurred in psychology that a quantitative concept was defined by an exact description of tests but that the expectation of finding laws connecting the values thus measured with values of other concepts was not fulfilled; then the concept was finally discarded as not fruitful. If it is a question of an explication of a prescientific concept, then a situation of the kind described, where we do not succeed in finding an adequate quantitative explicatum, ought not to discourage us altogether from trying an explication. It may be possible to find an adequate comparative explicatum. Let us show this by a fictitious example. The experience leading to the concept Temperature was first a comparative one; it was found that, if x is warmer than y (in the prescientific sense) and we bring a body of mercury first in contact with x and later with y, then it has at the first occasion a greater volume than at the second. By a certain device it was made possible to measure the small differences in the volume of the mercury; and that was taken as basis for the quantitative concept Temperature. Now let us assume fictitiously that we did not find technical means for measuring the differences in the volume of the mercury, although we were able to observe whether the mercury expands or contracts. In this case we should have no basis for a quantitative concept Temperature, but it would still be possible to define the comparative concept Warmer* with reference to an expansion of the mercury. This scientific concept Warmer* could then be taken as explicatum for the prescientific concept Warmer. Here, in the fictitious case, the concept Warmer* would be of greater importance than it is in actual physics, because it would be the only explicatum. Note that Warmer* here is essentially the same concept as Warmer* in the earlier discussion but that there is a difference in the form of the two definitions. In the former case we defined Warmer* in terms of higher temperature, hence with the help of a quantitative concept; here, in the fictitious case, it is defined with reference to the comparative concept of the expansion of mercury without the use of quantitative concepts. The distinction between these two ways of defining a comparative concept, the quantitative way and the purely comparative, that is, nonquantitative, way, will be of importance later when we discuss the comparative concept of confirmation.

To make a weaker fictitious assumption, suppose that the volume dif-

ferences could be measured and hence the quantitative concept Temperature could be defined but that—this is the fictitious feature—no important laws containing this concept had been found. In this case the concept would be discarded as not fruitful. And hence in this case likewise the comparative concept Warmer* would be taken as the only explicatum for Warmer.

Later, when we discuss the problem of explication for the concept of confirmation, we shall distinguish three concepts, the classificatory, the comparative, and the quantitative concept of confirmation. They are analogous to the concepts Warm, Warmer, and Temperature; thus the results of the present discussion will then be utilized.

§ 6. Formalization and Interpretation

> The axiomatic method consists of two phases, formalization and interpretation. The formalization of a theory consists in the construction of an axiom system. This is a semiformal system; the axiomatic terms are left uninterpreted, while some logical terms are taken with their customary meanings. The interpretation of an axiom system is given by rules which determine the meanings of the axiomatic terms. As an illustration for the distinction between the two phases, the difference between Peano's axiom system of arithmetic and the Frege-Russell system of arithmetic, which gives an interpretation, is explained.

The introduction of new concepts into the language of science—whether as explicata for prescientific concepts or independently—is sometimes done in two separate steps, formalization and interpretation. The procedure of separating these steps has steadily grown in importance during the last half-century. The two steps are the two phases of what is known as the axiomatic (or postulational) method in its modern form (as distinguished from its traditional form dating from Euclid). Frequently, the first step alone is already very useful, and sometimes considerable time passes until it is followed by the second step.

The formalization (or axiomatization) of a theory or of the concepts of a theory is here understood in the sense of the construction of a formal system, an *axiom system* (or postulate system) for that theory.

> We are not speaking here of a formal system in the strict sense, sometimes called a calculus (in the strict sense) or a syntactical system; in a system of this kind all rules are purely syntactical and all signs occurring are left entirely uninterpreted (see [Semantics] § 24). On the other hand, we are not speaking of axiom systems of the traditional kind, which are entirely interpreted. In the discussions of this book we are rather thinking of those semiformal, semi-interpreted systems which are constructed by contemporary authors, especially mathematicians, under the title of axiom systems (or postulate systems). In a system of this kind the axiomatic terms (for instance, in Hilbert's axiom sys-

tem of geometry the terms 'point', 'line', 'incidence', 'between', and others) remain uninterpreted, while for all or some of the logical terms occurring (e.g., 'not', 'or', 'every') and sometimes for certain arithmetical terms (e.g., 'one', 'two') their customary interpretation is—in most cases tacitly—presupposed. (For an explanation of the semiformal character of axiom systems see [Foundations] § 16.)

The *interpretation* of an axiom system consists in the interpretation of its primitive axiomatic terms. This interpretation is given by rules specifying the meanings which we intend to give to these terms; hence the rules are of a semantical nature. (They are sometimes called correlative definitions (Reichenbach's "Zuordnungsdefinitionen") or epistemic correlations (Northrop).) Sometimes the interpretation of a term can be given in the simple form of an explicit definition; this definition may be regarded as a semantical rule which states that the term in question is to have the same meaning as a certain compound expression consisting of terms whose meanings are presupposed as known.

For our later discussions on probability it will be of great importance to recognize clearly the character of the axiomatic method and especially the distinction between formalization and interpretation. Some authors believe they have given a solution of the problem of probability, in our terminology, an explication for probability, by merely constructing an axiom system for probability without giving an interpretation; for a genuine explication, however, an interpretation is essential. We shall now illustrate the axiomatic method and the distinction between its two phases by taking as an example the arithmetic of natural numbers. The prescientific terms of this field are the numerals 'one', 'two', etc. (or the corresponding figures) and terms for arithmetical operations like 'plus' (previously 'and'), 'times', etc., as they are used in everyday language for counting things and for calculating with numbers applied to things. Preliminary steps toward a systematization of the theory and an explication of the terms have been made for several thousand years in the form of rules of calculation. The first axiom system for arithmetic which satisfies modern requirements as to the exactness of formulation is the famous axiom system of *G. Peano*. This system takes as primitive axiomatic terms 'o', 'number', and 'successor'. It consists of five axioms, among them: 'o is a number' and 'the successor of a number is a number'. On the basis of the primitive terms mentioned, terms for the ordinary arithmetical operations can be introduced by recursive definitions. On the basis of the axioms and the recursive definitions, the ordinary theorems of elementary arithmetic can be proved. In this procedure the primitive terms mentioned and the terms introduced on their basis remain uninterpreted. It is only for di-

dactic, psychological reasons that not arbitrarily chosen symbols are taken as primitive terms but customary signs or words. Their well-known meanings facilitate the manipulations of the signs in the deductions, but these deductions are formal in the sense that they do not make use of the meanings of the axiomatic terms at any point.   ·

Peano's axiom system, by furnishing the customary formulas of arithmetic, achieves in this field all that is to be required from the point of view of formal mathematics. However, it does not yet achieve an explication of the arithmetical terms 'one', 'two', 'plus', etc. In order to do this, an interpretation must be given for the semiformal axiom system. There is an infinite number of true interpretations for this system, that is, of sets of entities fulfilling the axioms, or, as one usually says, of models for the system. One of them is the set of natural numbers as we use them in everyday life. But it can be shown that all sets of any entities exhibiting the same structure as the set of natural numbers in their order of magnitude—in Russell's terminology, all progressions—are likewise models of Peano's system. From the point of view of the formal system, no distinction is made between these infinitely many models. However, in order to state the one interpretation we are aiming at, we have to give an explication for the terms 'one', 'two', etc., as they are meant when we apply them in everyday life.

The first exact explications for the ordinary arithmetical terms have been given by G. Frege and later in a similar way by Bertrand Russell. Both Frege and Russell give explicata for the arithmetical concepts by explicit definitions on the basis of a purely logical system whose primitive terms are presupposed as interpreted. On the basis of this interpretation of the arithmetical terms, Peano's axioms become provable theorems in logic. It is a historically and psychologically surprising fact that this explication was such a difficult task and was achieved so late, although the explicanda, the elementary concepts of arithmetic, are understood and correctly applied by every child and have been successfully applied and to some extent also systematized for thousands of years.

It is important to see clearly the difference between Peano's and Frege's systems of arithmetic. Peano's system, as mentioned, does not go beyond the boundaries of formal mathematics. Only Frege's system enables us to apply the arithmetical concepts in the description of facts; it enables us to transform a sentence like 'the number of fingers on my right hand is 5' into a form which does not contain any arithmetical terms. Peano's system contains likewise the term '5', but only as an uninterpreted symbol. It enables us to derive formulas like '3 + 2 = 5', but it does not tell us

how to understand the term '5' when it occurs in a factual sentence like that about the fingers. Only Frege's system enables us to understand sentences of this kind, that is to say, to know what we have to do in order to find out whether the sentence is true or not.

The result of this discussion is, in general terms, the following. As soon as we go over from the field of formal mathematics to that of knowledge about the facts of nature, in other words, to empirical science, which includes applied mathematics, we need more than a mere calculus or axiom system; an interpretation must be added to the system.

> Concerning the arithmetical systems of Peano, Frege, and Russell: G. Peano, *Arithmetices principia* (1889); G. Frege, *Grundlagen der Arithmetik* (1884); *Grundgesetze der Arithmetik* (2 vols.; 1893, 1903); Bertrand Russell, *The principles of mathematics* (1903); with A. N. Whitehead, [Princ. Math.]. For a discussion of the distinction between Peano's arithmetic and that of Frege and Russell see Russell, *Introduction to mathematical philosophy* (1918), chaps. 1 and 2; and Carnap [Foundations] §§ 17 ff.